In the top view, b_2 will move to a point b_3 along the line *rs*, drawn through b_2 and parallel to *xy*, thus keeping its distance from the path of *a*, viz. b_2o constant. *rs* is the locus or path of the end *B* in the top view. The point b_3 lies on the projector through b'_3. The new top view ab_3 is shorter than ab_2 (i.e. *AB*) and makes an angle β with *xy*. β is greater than ø.

Here also we find that, as long as the inclination of *AB* with the V.P. does not change, even when it becomes inclined to the H.P.

(i) its length in the front view, viz. $a'b'_2$ remains constant; and

(ii) the distance between the paths of its ends in the top view, viz. b_2o remains constant.

Hence, when a line is inclined to both the planes, its projections are shorter than the true length and inclined to xy at angles greater than the true inclinations. These angles viz. α *and* β *are called apparent angles of inclination.*

10-6. PROJECTIONS OF LINES INCLINED TO BOTH THE PLANES

From Art. 10-5(a) above, we find that as long as the inclination of *AB* with the H.P. is constant

(i) its length in the top view, viz. *ab* remains constant, and

(ii) in the front view, the distance between the loci of its ends, viz. *b'o* remains constant.

In other words if

(i) its length in the top view is equal to *ab*, and

(ii) the distance between the paths of its ends in the front view is equal to *b'o*, the inclination of *AB* with the H.P. will be equal to θ.

Similarly, from Art. 10-5(b) above, we find that as long as the inclination of *AB* with the V.P. is constant

(i) its length in the front view, viz. $a'b'_2$ remains constant, and

(ii) in the top view, the distance between the loci of its ends, viz. b_2o remains constant.

The reverse of this is also true, viz.

(i) if its length in the front view is equal to $a'b'_2$, and

(ii) the distance between the paths of its ends in the top view is equal to b_2o, the inclination of *AB* with the V.P. will be equal to ø.

Combining the above two findings, we conclude that when *AB* is inclined at θ to the H.P. and at ø to the V.P.

(i) its lengths in the top view and the front view will be equal to ab_2 and $a'b'_2$ respectively, and

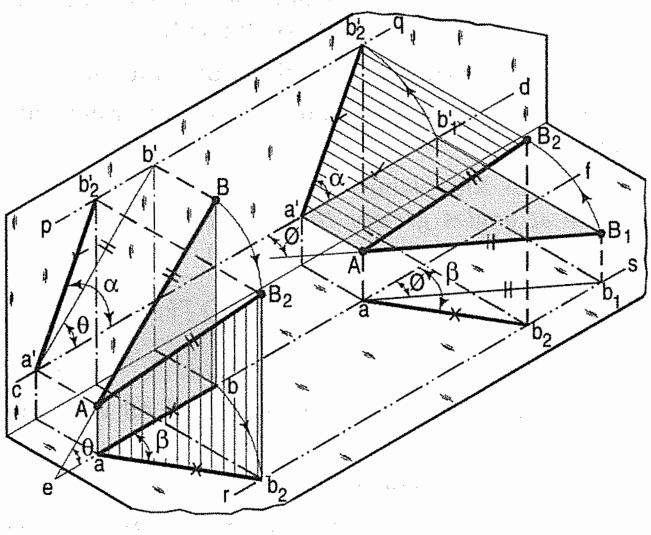
(ii) the distances between the paths of its ends in the front view and the top view will be equal to b'_2o and b_2o respectively.

The two lengths when arranged with their ends in their respective paths and in projections with each other will be the projections of the line *AB*, as illustrated in problem 10-4.

Problem 10-4. *Given the line AB, its inclinations θ with the H.P. and ø with the V.P. and the position of one end A. To draw its projections.*

Mark the front view a' and the top view *a* according to the given position of *A* (fig. 10-12).

Let us first determine the lengths of *AB* in the top view and the front view and the paths of its ends in the front view and the top view.

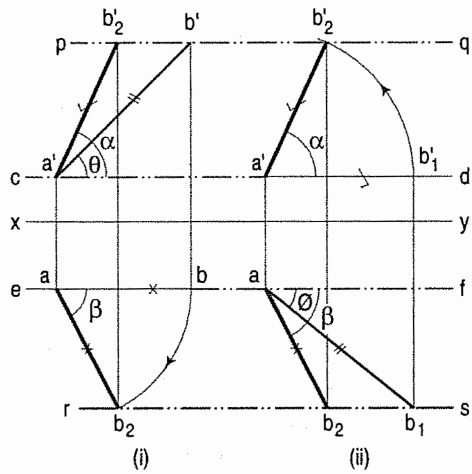


(i) (ii)

FIG. 10-11

(1) Assume *AB* to be parallel to the V.P. and inclined at θ to the H.P. *AB* is shown in the pictorial view as a side of the trapezoid *ABba* [fig. 10-11(i)]. Draw the front view *a'b'* equal to *AB* [fig. 10-12(i)] and inclined at θ to *xy*. Project the top view *ab* parallel to *xy*. Through *a'* and *b'*, draw lines *cd* and *pq* respectively parallel to *xy*. *ab* is the length of *AB* in the top view and, *cd* and *pq* are the paths of *A* and *B* respectively in the front view.

(2) Again, assume AB_1 (equal to *AB*) to be parallel to the H.P. and inclined at ø to the V.P. In the pictorial view [fig. 10-11(ii)], AB_1 is shown as a side of the trapezoid $AB_1b'_1a'$. Draw the top view ab_1 equal to *AB* [fig. 10-12(ii)] and inclined at ø to *xy*. Project the front view $a'b'_1$ parallel to *xy*. Through *a* and b_1, draw lines *ef* and *rs* respectively parallel to *xy*. $a'b'_1$ is the length of *AB* in the front view and, *ef* and *rs* are the paths of *A* and *B* respectively in the top view.



(i) (ii)

FIG. 10-12

We may now arrange

(i) *ab* (the length in the top view) between its paths *ef* and *rs*, and

(ii) $a'b'_1$ (the length in the front view) between the paths *cd* and *pq*, keeping them in projection with each other, in one of the following *two* ways:

(a) In case (1) [fig. 10-11(i)], if the side *Bb* is turned about *Aa*, so that *b* comes on the path *rs*, the line *AB* will become inclined at ø to the V.P. Therefore, with *a* as centre [fig. 10-12(i)] and radius equal to *ab*, draw an arc cutting *rs* at a point b_2. Project b_2 to b'_2 on the path *pq*.

Draw lines joining a with b_2, and a' with b'_2. ab_2 and $a'b'_2$ are the required projections. Check that $a'b'_2 = a'b'_1$.

(b) Similarly, in case (2) [fig. 10-11(ii)], if the side $B_1b'_1$ is turned about Aa' till b_1' is on the path pq, the line AB_1 will become inclined at θ to the H.P.

Hence, with a' as centre [fig. 10-12(ii)] and radius equal to $a'b'_1$, draw an arc cutting pq at a point b'_2. Project b'_2 to b_2 in the top view on the path rs.

Draw lines joining a with b_2, and a' with b'_2. ab_2 and $a'b'_2$ are the required projections. Check that $ab_2 = ab$.

Fig. 10-13 shows (in pictorial and orthographic views) the projections obtained with both the above steps combined in one figure and as described below.

First, determine

(i) the length ab in the top view and the path pq in the front view and

(ii) the length $a'b'_1$ in the front view and the path rs in the top view.

Then, with a as centre and radius equal to ab, draw an arc cutting rs at a point b_2. With a' as centre and radius equal to $a'b'_1$, draw an arc cutting pq at a point b'_2.

Draw lines joining a with b_2 and a' with b'_2. ab_2 and $a'b'_2$ are the required projections. Check that b_2 and b'_2 lie on the same projector.

It is quite evident from the figure that the apparent angles of inclination α and β are greater than the true inclinations θ and ø respectively.

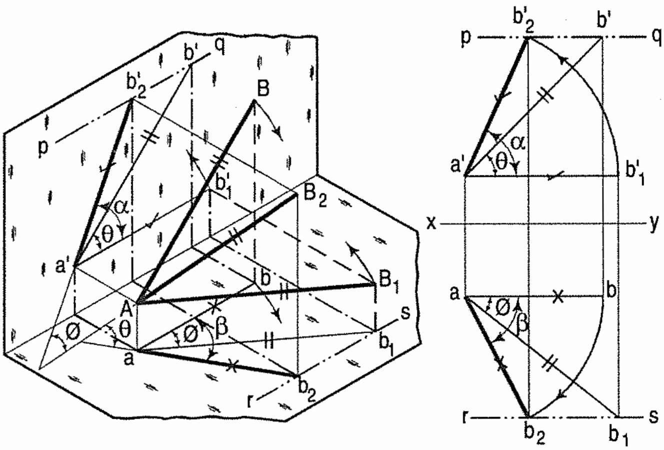


FIG. 10-13

10-7. LINE CONTAINED BY A PLANE PERPENDICULAR TO BOTH THE REFERENCE PLANES

As the two reference planes are at right angles to each other, the sum total of the inclinations of a line with the two planes, viz. θ and ø can never be more than 90°. When θ + ø = 90°, the line will be contained by a third plane called the profile plane, perpendicular to both the H.P. and the V.P.

A line *EF* (fig. 10-14), is inclined at θ to the H.P. and at ø [equal to (90° – θ)] to the V.P. The line is thus contained by the profile plane marked P.P.

The front view *e'f'* and the top view *ef* are both perpendicular to *xy* and shorter than *EF*.

Therefore, when a line is inclined to both the reference planes and contained by a plane perpendicular to them, i.e. when the sum of its inclinations with the H.P. and the V.P. is 90°, its projections are perpendicular to *xy* and shorter than the true length.

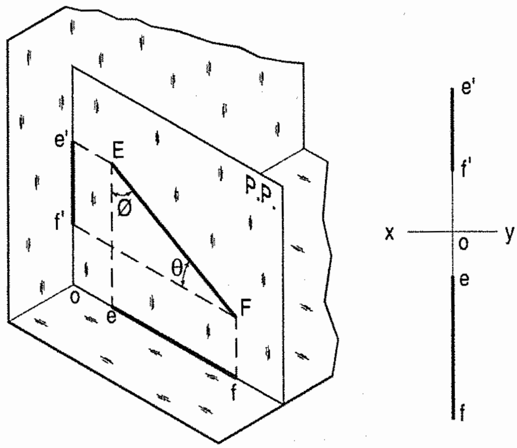


FIG. 10-14

10-8. TRUE LENGTH OF A STRAIGHT LINE AND ITS INCLINATIONS WITH THE REFERENCE PLANES

When projections of a line are given, its true length and inclinations with the planes are determined by the application of the following rule:

When a line is parallel to a plane, its projection on that plane will show its true length and the true inclination with the other plane.

The line may be made parallel to a plane, and its true length obtained by any one of the following *three* methods:

Method I:

Making each view parallel to the reference line and projecting the other view from it. This is the exact reversal of the processes adopted in Art. 10-5 for obtaining the projections.

Method II:

Rotating the line about its projections till it lies in the H.P. or in the V.P.

Method III:

Projecting the views on auxiliary planes parallel to each view.

(This method will be dealt with in chapter 11).

The following problem shows the application of the first two methods and problem 10-29 and problem 10-31 show application of third method.

Problem 10-5. *The top view ab and the front view a'b' of a line AB are given. To determine its true length and the inclinations with the H.P. and the V.P.*

Method I:

Fig. 10-15(i) shows *AB* the line, *a'b'* its front view and *ab* its top view. If the trapezoid *ABba* is turned about *Aa* as axis, so that *AB* becomes parallel to the V.P., in the top view, *b* will move along an arc drawn with centre *a* and radius equal to *ab*, to b_1, so that ab_1 is parallel to *xy*. In the front view, *b'* will move along its locus *pq*, to a point b'_1 on the projector through b_1.

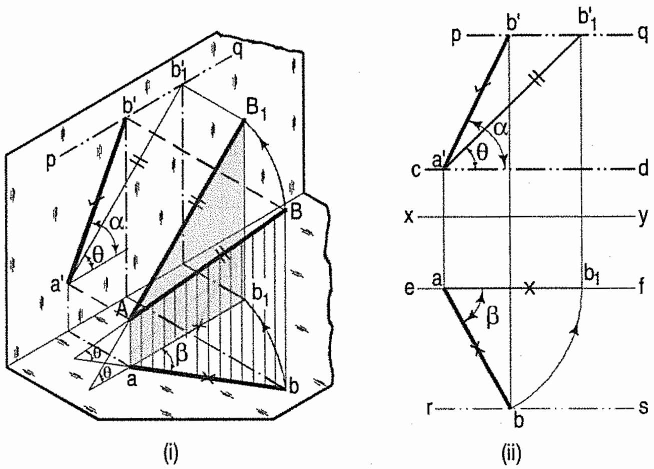


FIG. 10-15

(i) Therefore, with centre *a* and radius equal to *ab* [fig. 10-15(ii)], draw an arc to cut *ef* at b_1.

(ii) Draw a projector through b_1 to cut *pq* (the path of *b*') at b'_1.

(iii) Draw the line $a'b'_1$ which is the true length of *AB*. The angle θ, which it makes with *xy* is the inclination of *AB* with the H.P.

Again, in fig. 10-16(i) *AB* is shown as a side of a trapezoid *ABb'a'*. If the trapezoid is turned about *Aa'* as axis so that *AB* is parallel to the H.P., the new top view will show its true length and true inclination with the V.P.

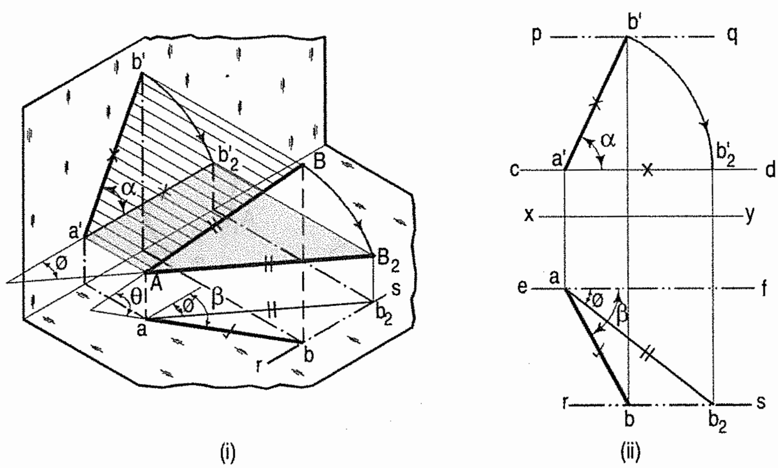


FIG. 10-16

(i) With *a*' as centre and radius equal to *a'b*' [fig. 10-16(ii)], draw an arc to cut *cd* at b'_2.

(ii) Draw a projector through b'_2 to cut *rs* (the path of *b*) at b_2.

(iii) Draw the line ab_2, which is the true length of *AB*. The angle ø which it makes with *xy* is the inclination of *AB* with the V.P.

Fig. 10-17(i) shows the above two steps combined in one figure.

The same results will be obtained by keeping the end B fixed and turning the end A [fig. 10-17(ii)], as explained below.

(i) With centre b and radius equal to ba, draw an arc cutting rs at a_1 (thus making ba parallel to xy).

(ii) Project a_1 to a'_1 on cd (the path of a') a'_1b' is the true length and θ is the true inclination of AB with the H.P.

(iii) Similarly, with centre b' and radius equal to $b'a'$, draw an arc cutting pq at a_2'.

(iv) Project a_2' to a_2 on ef (the path of a). a_2b is the true length and $ø$ is the true inclination of AB with the V.P.

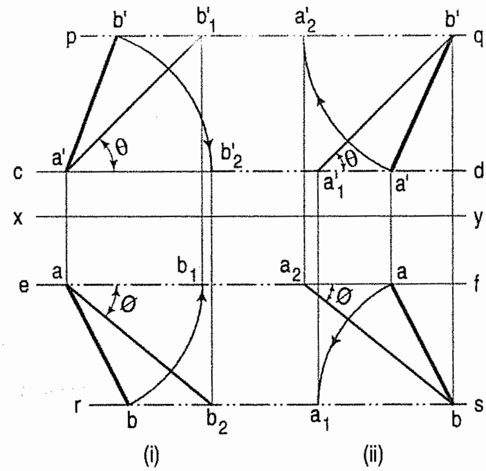


FIG. 10-17

Method II:

Referring to the pictorial view in fig. 10-18(i) we find that AB is the line, ab its top view and $a'b'$ its front view.

In the trapezoid $ABb'a'$ (i) $a'A$ and $b'B$ are both perpendicular to $a'b'$ and are respectively equal to ao_1 and bo_2 (the distances of a and b from xy in the top view), and (ii) the angle between AB and $a'b'$ is the angle of inclination $ø$ of AB with the V.P.

Assume that this trapezoid is rotated about $a'b'$, till it lies in the V.P.

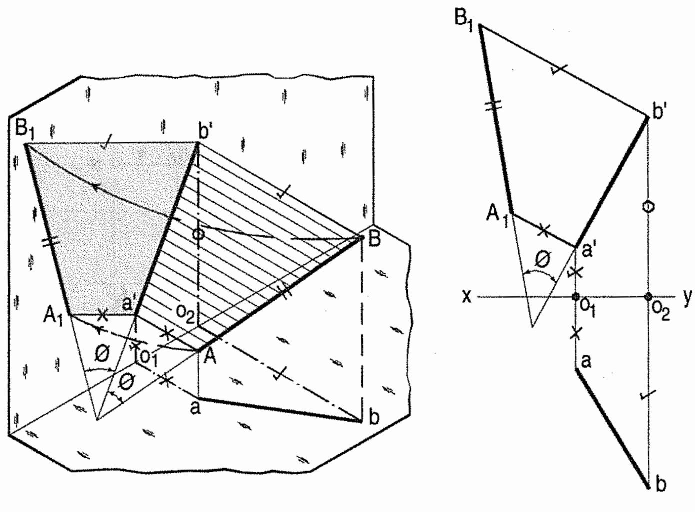


FIG. 10-18

In the orthographic view [fig. 10-18(ii)], this trapezoid is obtained by drawing perpendiculars to $a'b'$, viz. $a'A_1$ (equal to ao_1) and $b'B_1$ (equal to bo_2) and then joining A_1 with B_1. The line A_1B_1 is the true length of AB and its inclination $ø$ with a' b' is the inclination of AB with the V.P.

Similarly, in trapezoid *ABba* in fig. 10-19(i), *AB* is the line and *ab* its top view. *Aa* and *Bb* are both perpendicular to *ab* and are respectively equal to $a'o_1$ and $b'o_2$ (the distances of *a'* and *b'* from *xy* in the front view). The angle θ between *AB* and *ab* is the inclination of *AB* with the H.P.

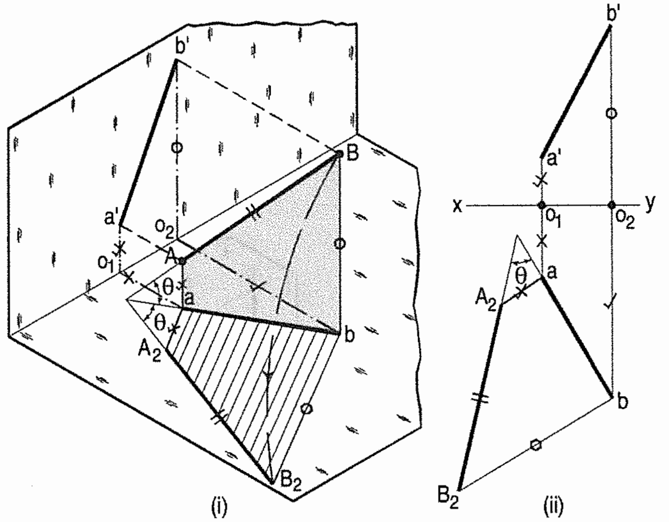


FIG. 10-19

This figure may now be assumed to be rotated about *ab* as axis, so that it lies in the H.P.

In the orthographic view [fig. 10-19(ii)], this trapezoid is obtained by erecting perpendiculars to *ab*, viz. aA_2 equal to $a'o_1$ and bB_2 equal to $b'o_2$ and joining A_2 with B_2. The line A_2B_2 is the true length of *AB* and its inclination θ with *ab* is the inclination of *AB* with the H.P.

Note: The perpendiculars on *ab* or *a'b'* can also be drawn on its other side assuming the trapezoid to be rotated in the opposite direction.

10-9. TRACES OF A LINE

When a line is inclined to a plane, it will meet that plane, produced if necessary. The point in which the line or line-produced meets the plane is called its *trace*.

The point of intersection of the line with the H.P. is called the *horizontal trace*, usually denoted as H.T. and that with the V.P. is called the *vertical trace* or V.T.

Refer to fig. 10-20.

(i) A line *AB* is parallel to the H.P. and the V.P. It has no trace.

(ii) A line *CD* is inclined to the H.P. and parallel to the V.P. It has only the H.T. but no V.T.

(iii) A line *EF* is inclined to the V.P. and parallel to the H.P. It has only the V.T. but no H.T.

Thus, when a line is parallel to a plane it has no trace upon that plane.

Refer to fig. 10-21.

(i) A line *PQ* is perpendicular to the H.P. Its H.T. coincides with its top view which is a point. It has no V.T.

(ii) A line *RS* is perpendicular to the V.P. Its V.T. coincides with its front view which is a point. It has no H.T.

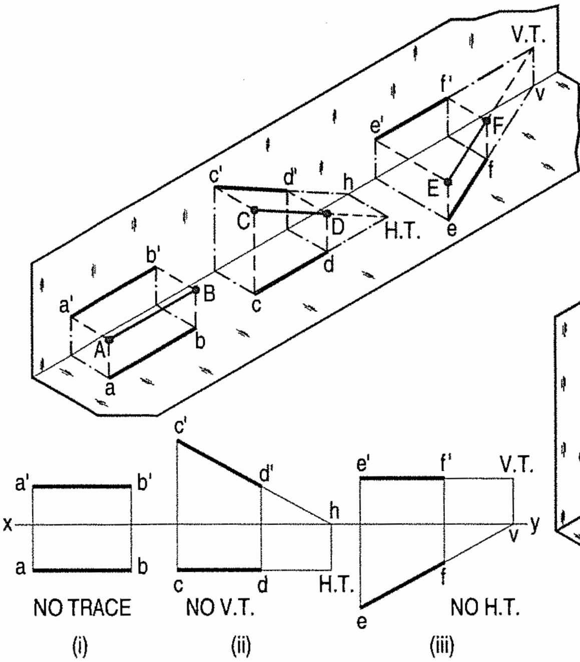


FIG. 10-20

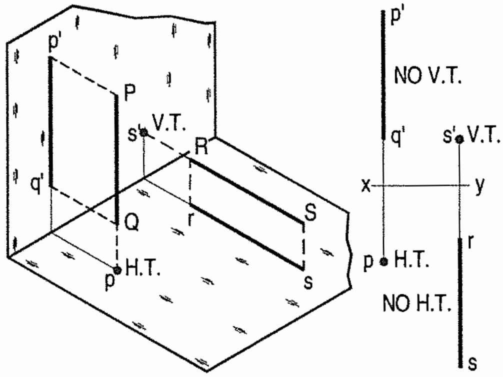


FIG. 10-21

Hence, when a line is perpendicular to a plane, its trace on that plane coincides with its projection on that plane. It has no trace on the other plane.

Refer to fig. 10-22.

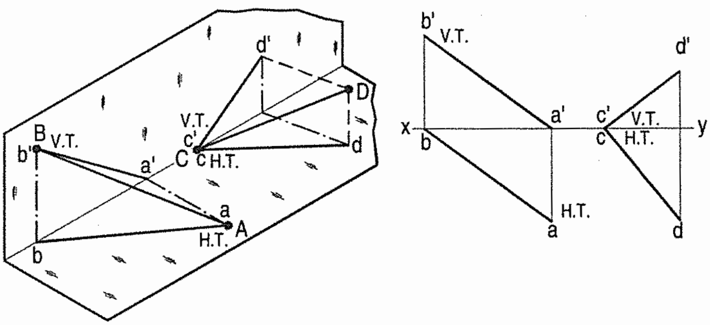


FIG. 10-22

(i) A line *AB* has its end *A* in the H.P. and the end *B* in the V.P. Its H.T. coincides with *a* the top view of *A* and the V.T. coincides with *b*' the front view of *B*.

(ii) A line *CD* has its end *C* in both the H.P. and the V.P. Its H.T. and V.T. coincide with *c* and *c*' (the projections of *C*) in *xy*.

Hence, when a line has an end in a plane, its trace upon that plane coincides with the projection of that end on that plane.

10-10. METHODS OF DETERMINING TRACES OF A LINE

Method I:

Fig. 10-23(i) shows a line *AB* inclined to both the reference planes. Its end *A* is in the H.P. and *B* is in the V.P.

a'b' and *ab* are the front view and the top view respectively [fig. 10-23(ii)].

The H.T. of the line is on the projector through *a'* and coincides with *a*. The V.T. is on the projector through *b* and coincides with *b'*.

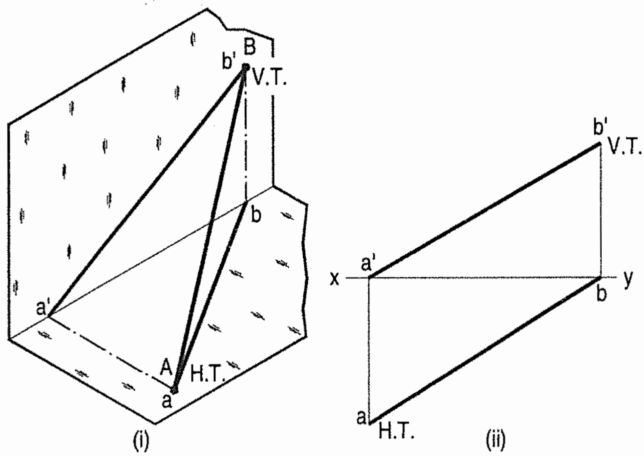


FIG. 10-23

Let us now assume that *AB* is shortened from both its ends, its inclination with the planes remaining constant. The H.T. and V.T. of the new line *CD* are still the same as can be seen clearly in fig. 10-24(i).

c'd' and *cd* are the projections of *CD* [fig. 10-24(ii)]. Its traces may be determined as described below.

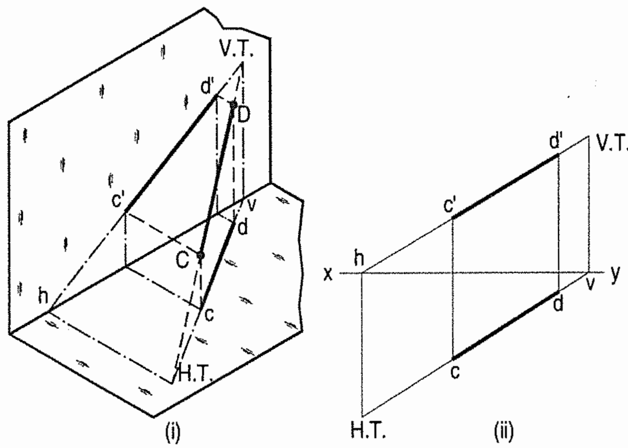


FIG. 10-24

(i) Produce the front view *c'd'* to meet *xy* at a point *h*.

(ii) Through *h*, draw a projector to meet the top view *cd*-produced, at the H.T. of the line.

(iii) Similarly, produce the top view *cd* to meet *xy* at a point *v*.

(iv) Through *v*, draw a projector to meet the front view *c'd'*-produced, at the V.T. of the line.

Method II:

c'd' and *cd* are the projections of the line *CD* [fig. 10-25(ii)]. Determine the true length C_1D_1 from the front view *c'd'* by trapezoid method. The point of intersection between *c'd'*-produced and C_1D_1-produced is the V.T. of the line.

Similarly, determine the true length C_2D_2 from the top view *cd*. Produce them to intersect at the H.T. of the line.

The above is quite evident from the pictorial view shown in fig. 10-25(i).

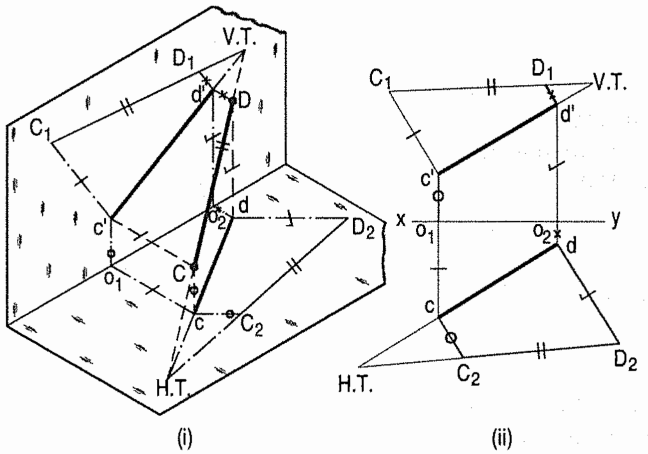


FIG. 10-25

10-11. TRACES OF A LINE, THE PROJECTIONS OF WHICH ARE PERPENDICULAR TO *xy*

When the projections of a line are perpendicular to *xy*, i.e. when the sum of its inclinations with the two principal planes of projection is 90°, it is not possible to find the traces by the first method. Method II must, therefore, be adopted as shown in fig. 10-26.

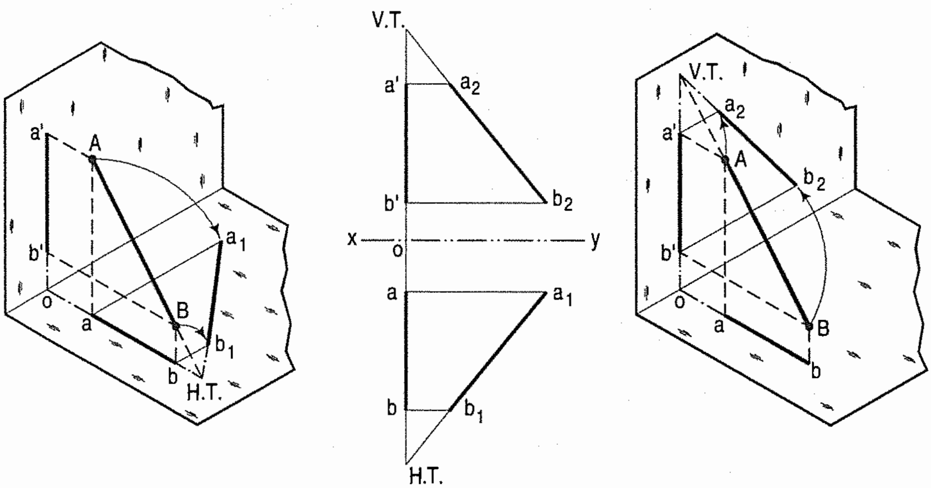


FIG. 10-26

10-12. POSITIONS OF TRACES OF A LINE

Although the line may be situated in the third quadrant, its both traces may be above or below *xy*, as shown in problem 10-6 and in fig. 10-27 and fig. 10-28. When a line intersects a plane, its traces on that plane will be contained by its projection on that plane as shown in problem 10-7.

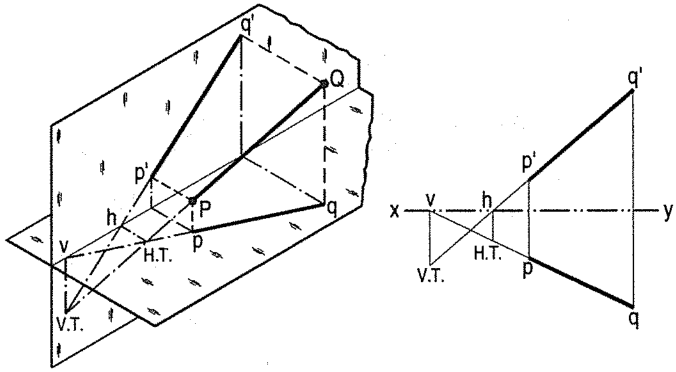


FIG. 10-27

Problem 10-6. *Projections of a line PQ are given. Determine the positions of its traces.*

Let *pq* and *p'q'* be the projections of *PQ* (fig. 10-27 and fig. 10-28).

(i) Produce the top view *pq* to meet *xy* at *v*. Draw a projector through *v* to meet the front view *p'q'*-produced at the V.T.

(ii) Through *h*, the point of intersection between *p'q'*-produced and *xy*, draw a projector to meet the top view *pq*-produced at the H.T.

Note that in fig. 10-27, the traces are below *xy* while in fig. 10-28 they are above it.

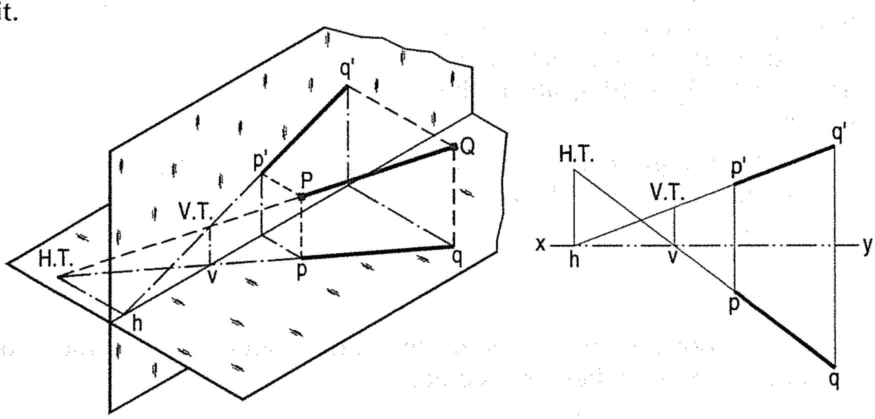


FIG. 10-28

Problem 10-7. *A point A is 50 mm below the H.P. and 12 mm behind the V.P. A point B is 10 mm above the H.P. and 25 mm in front of the V.P. The distance between the projectors of A and B is 40 mm. Determine the traces of the line joining A and B.*

Draw the projections *ab* and *a'b'* of the line *AB*.

Method I: (fig. 10-29):

(i) Through *v*, the point of intersection between *ab* and *xy*, draw a projector to meet *a'b'* at the V.T. of the line.

(ii) Similarly, through *h*, the point of intersection between *a'b'* and *xy*, draw a projector to cut *ab* at the H.T. of the line.

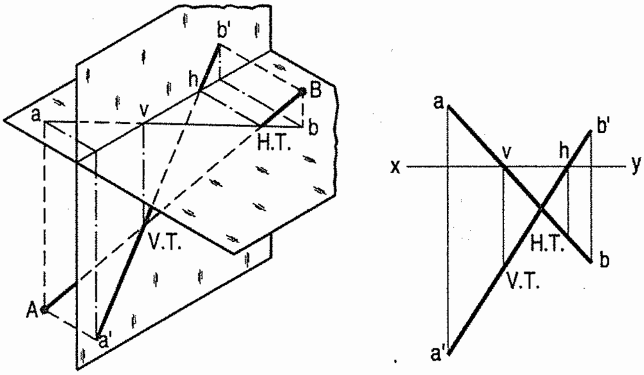


FIG. 10-29

Method II: (fig. 10-30):

At the ends a' and b', draw perpendiculars to $a'b'$, viz. $a'A_1$ equal to ao_1 and $b'B_1$ equal to bo_2 on its opposite sides (as a and b are on opposite sides of xy).

Draw the line A_1B_1 intersecting $a'b'$ at the V.T. of the line.

Similarly, at the ends a and b, draw perpendiculars to ab, viz. aA_2 equal to $a'o_1$ and bB_2 equal to $b'o_2$, on its opposite sides (as a' and b' are on opposite sides of xy). Join A_2 with B_2 cutting ab at the H.T. of the line.

Note that $A_1B_1 = A_2B_2 = AB$ and that θ and ø are the inclinations of AB with the H.P. and the V.P. respectively.

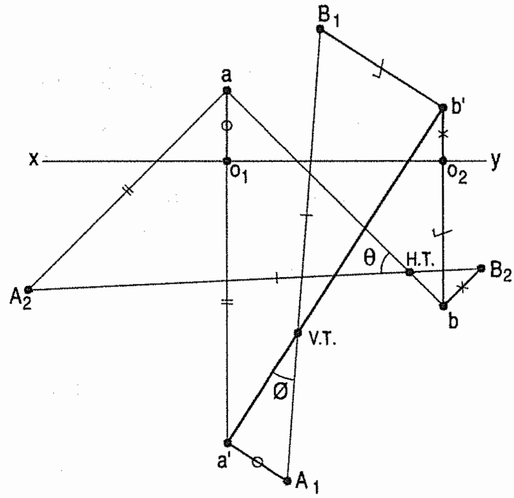


FIG. 10-30

10-13. ADDITIONAL ILLUSTRATIVE PROBLEMS

In the following problems, the ends of the lines should be assumed to be in the first quadrant, unless otherwise stated.

Problem 10-8. (fig. 10-31): *A line AB, 50 mm long, has its end A in both the H.P. and the V.P. It is inclined at 30° to the H.P. and at 45° to the V.P. Draw its projections.*

As the end A is in both the planes, its top view and the front view will coincide in xy.

(i) Assuming AB to be parallel to the V.P. and inclined at θ (equal to 30°) to the H.P., draw its front view ab' (equal to AB) and project the top view ab.

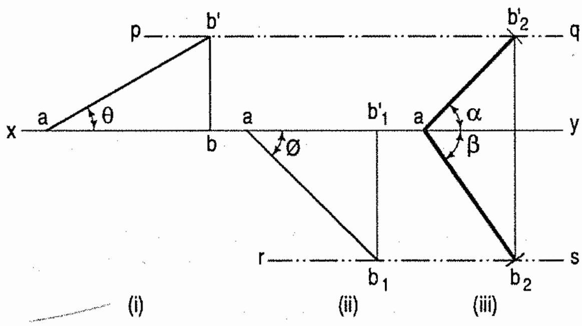


FIG. 10-31

(ii) Again assuming *AB* to be parallel to the H.P. and inclined at ø (equal to 45°) to the V.P., draw its top view ab_1 (equal to *AB*). Project the front view ab'_1.

ab and ab'_1 are the lengths of *AB* in the top view and the front view respectively, and *pq* and *rs* are the loci of the end *B* in the front view and the top view respectively.

(iii) With *a* as centre and radius equal to ab'_1, draw an arc cutting *pq* in b'_2. With the same centre and radius equal to *ab*, draw an arc cutting *rs* in b_2.

Draw lines joining *a* with b'_2 and b_2. ab'_2 and ab_2 are the required projections.

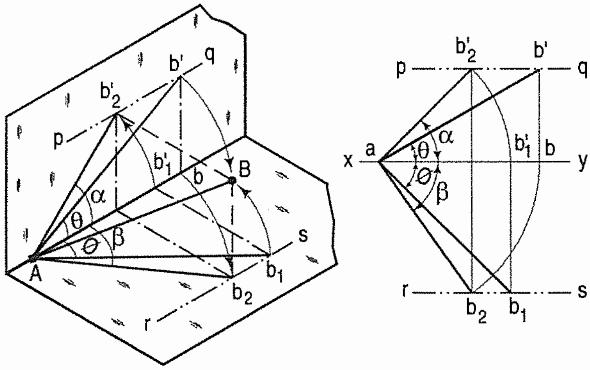


FIG. 10-32

Fig. 10-32 shows in pictorial and orthographic views, the solution obtained with all the above steps combined in one figure only.

Problem 10-9. (fig. 10-33): *A line PQ 75 mm long, has its end P in the V.P. and the end Q in the H.P. The line is inclined at 30° to the H.P. and at 60° to the V.P. Draw its projections.*

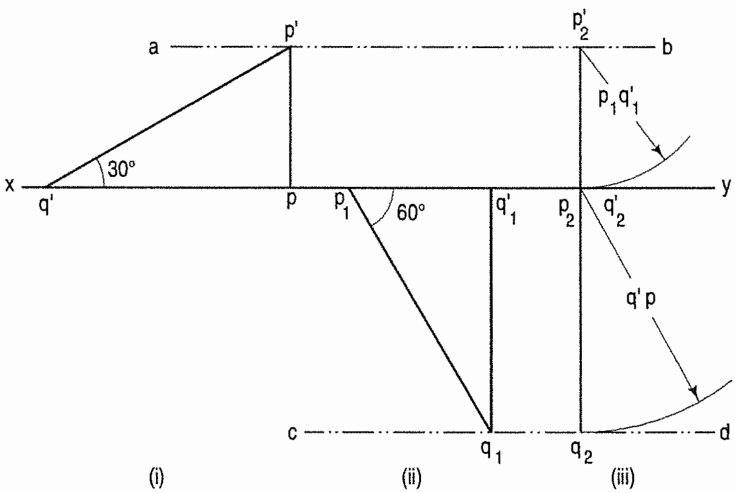


FIG. 10-33

The top view of *P* and the front view of *Q* will be in *xy*. As shown in the previous problem, determine

(i) the length of PQ in the top view, viz. $q'p$ and the path ab of the end P in the front view;

(ii) the length $p_1q'_1$ in the front view and the path cd of the end Q in the top view.

(iii) Mark any point p_2 (the top view of P) in xy and project its front view p'_2 in ab.

(iv) With p'_2 as centre and radius equal to $p_1q'_1$, draw an arc cutting xy in q'_2. It coincides with p_2.

(v) With p_2 as centre and radius equal to $q'p$, draw an arc cutting cd in q_2. p_2q_2 and $p'_2q'_2$ are the required projections. They lie in a line perpendicular to xy because the sum of the two inclinations is equal to 90°.

Problem 10-10. (fig. 10-34): *A line PQ 100 mm long, is inclined at 30° to the H.P. and at 45° to the V.P. Its mid-point is in the V.P. and 20 mm above the H.P. Draw its projections, if its end P is in the third quadrant and Q in the first quadrant.*

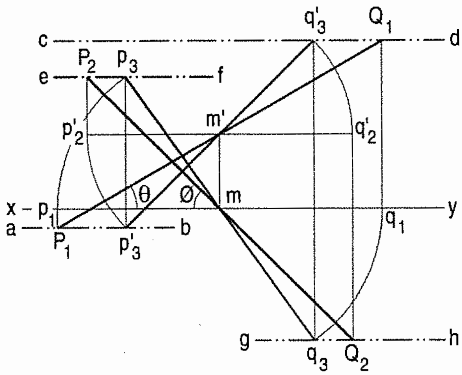


FIG. 10-34

The front view and the top view of P will be below and above xy respectively, while those of Q will be above and below xy respectively.

(i) Mark m, the top view of the mid-point in xy and project its front view m', 20 mm above xy.

(ii) Through m', draw a line making an angle θ (equal to 30°) with xy and with the same point as centre and radius equal to $\frac{1}{2}$ PQ, cut it at P_1 below xy and at Q_1 above xy. Project P_1Q_1 to p_1 q_1 on xy. p_1q_1 is the length of PQ in the top view. ab and cd are the paths of P and Q respectively in the front view.

(iii) Similarly, through m, draw a line making angle ø (equal to 45°) with xy and cut it with the same radius at P_2 above xy and at Q_2 below it.

(iv) Project P_2Q_2 to $p'_2q'_2$ on the horizontal line through m'. $p'_2q'_2$ is the length of PQ in the front view and ef and gh are the paths of P and Q respectively in the top view.

(v) With m as centre and radius equal to mp_1 or mq_1, draw arcs cutting ef at p_3 and gh at q_3. With m' as centre and radius equal to $m'p'_2$ or $m'q'_2$, draw arcs cutting ab at p'_3 and cd at q'_3. p_3q_3 and $p'_3q'_3$ are the required projections.

Problem 10-11. (fig. 10-35): *The top view of a 75 mm long line AB measures 65 mm, while the length of its front view is 50 mm. Its one end A is in the H.P. and 12 mm in front of the V.P. Draw the projections of AB and determine its inclinations with the H.P. and the V.P.*

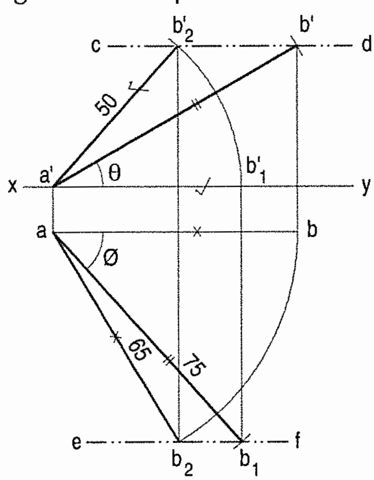


FIG. 10-35

(i) Mark the front view a' and the top view a of the given end A.

(ii) Assuming AB to be parallel to the V.P., draw a line ab equal to 65 mm and parallel to xy. With a' as centre and radius equal to 75 mm, draw an arc cutting the projector through b at b'. The line cd through b' and parallel to xy, is the locus of B in the front view and θ is the inclination of AB with the H.P.

(iii) Similarly, draw a line $a'b'_1$ in xy and equal to 50 mm. With a as centre and radius equal to AB, draw an arc cutting the projector through b'_1 at b_1. ef is the locus of B in the top view and $\varnothing$ is the inclination of AB with the V.P.

(iv) With a' as centre and radius equal to $a'b'_1$, draw an arc cutting cd in b'_2. With a as centre and radius equal to ab, draw an arc cutting ef in b_2. $a'b'_2$ and ab_2 are the required projections.

Problem 10-12. (fig. 10-36): *A line AB, 65 mm long, has its end A 20 mm above the H.P. and 25 mm in front of the V.P. The end B is 40 mm above the H.P. and 65 mm in front of the V.P. Draw the projections of AB and show its inclinations with the H.P. and the V.P.*

(i) As per given positions, draw the loci cd and gh of the end A, and ef and jk of the end B in the front view and the top view respectively.

(ii) Mark any point a (the top view of A) in gh and project it to a' on cd. With a' as centre and radius equal to 65 mm, draw an arc cutting ef in b'. Join a' with b'. θ, the inclination of $a'b'$ with xy, is the inclination of AB with the H.P. Project b' to b on gh. ab is the length of AB in the top view.

(iii) With a as centre and radius equal to 65 mm, draw an arc cutting jk in b_1. Join a with b_1. $\varnothing$, the inclination of ab_1 with xy, is the inclination of AB with the V.P. Project b_1 to b'_1 on cd. $a'b'_1$ is the length of AB in the front view.

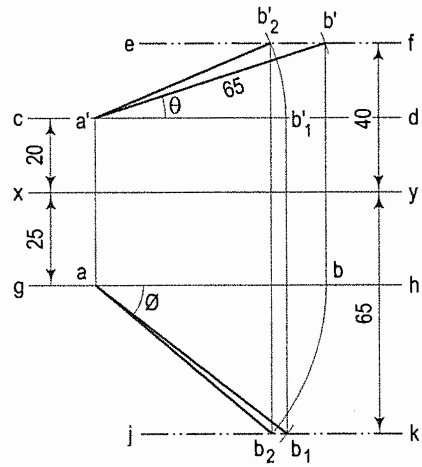


FIG. 10-36

Arrange ab and $a'b'_1$ between their respective paths as shown. $a'b'_2$ and ab_2 are the required projections of AB.

Problem 10-13. (fig. 10-37 and fig. 10-38): *The projectors of the ends of a line AB are 50 mm apart. The end A is 20 mm above the H.P. and 30 mm in front of the V.P. The end B is 10 mm below the H.P. and 40 mm behind the V.P. Determine the true length and traces of AB, and its inclinations with the two planes.*

Draw two projectors 50 mm apart. On one projector, mark the top view a and the front view a' of the end A. On the other, mark the top view b and the front view b' of the end B, as per given distances. ab and $a'b'$ are the projections of AB.

Determine the true length, traces and inclinations by any one of the following two methods:

Method I:

By making the line parallel to a plane (fig. 10-37):

(i) Keeping *a* fixed, turn *ab* to a position ab_1, thus making it parallel to *xy*. Project b_1 to b'_1 on the locus of *b'*. $a'b'_1$ is the true length of *AB* and θ is its true inclination with the H.P.

(ii) Similarly, turn *a'b'* to the position a'_1b' and project a'_1 to a_1 on the path of *a* (because the end *a* has been moved). a_1b is the true length of *AB* and ø is its inclination with the V.P.

Traces:

(i) Through *v* the point of intersection of the top view *ab* with *xy*, draw a projector to cut *a'b'* at the V.T.

(ii) Through *h* the point of intersection of the front view *a'b'* with *xy*, draw a projector to cut *ab* at the H.T. of the line.

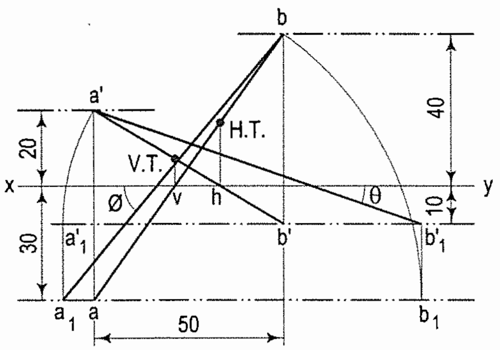


FIG. 10-37

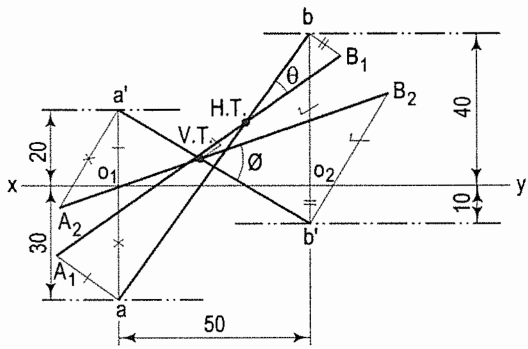


FIG. 10-38

Method II:

By rotating the line about its projections till it lies in H.P. or V.P. (fig. 10-38):

(i) At the ends *a* and *b* of the top view *ab*, draw perpendiculars to *ab*, viz. aA_1 equal to $a'o_1$ and bB_1 equal to $b'o_2$, on opposite sides of it (because *a'* and *b'* are on opposite sides of *xy*). A_1B_1 is the true length of *AB*. θ (its inclination with *ab*) is the inclination of *AB* with the H.P. and the point at which A_1B_1 intersects *ab* is the H.T. of *AB*.

(ii) Similarly, at the ends *a'* and *b'* of the front view *a'b'*, draw perpendiculars to *a'b'*, viz. $a'A_2$ equal to ao_1 and $b'B_2$ equal to bo_2, on opposite sides of it. A_2B_2 is the true length of *AB*. ø (its inclination with *a'b'*) is the inclination of *AB* with the V.P. and the point at which A_2B_2 intersects *a'b'* is the V.T. of *AB*.

Problem 10-14. (fig. 10-39): *A line AB, 90 mm long, is inclined at 45° to the H.P. and its top view makes an angle of 60° with the V.P. The end A is in the H.P. and 12 mm in front of the V.P. Draw its front view and find its true inclination with the V.P.*

(i) Mark *a* and *a'*, the projections of the end *A*.

(ii) Assuming *AB* to be parallel to the V.P. and inclined at 45° to the H.P., draw its front view *a'b'* equal to *AB* and making an angle of 45° with *xy*. Project *b'* to *b* so that *ab* the top view is parallel to *xy*. Keeping the end *a* fixed, turn the top view *ab* to a position ab_1 so that it makes an angle of 60° with *xy*. Project b_1 to b'_1 on the locus of *b'*. Join *a'* with b'_1. $a'b'_1$ is the front view of *AB*.

(iii) To find the true inclination with the V.P., draw an arc with *a* as centre and radius equal to *AB*, cutting the locus of b_1 in b_2. Join *a* with b_2. ø is the true inclination of *AB* with the V.P.

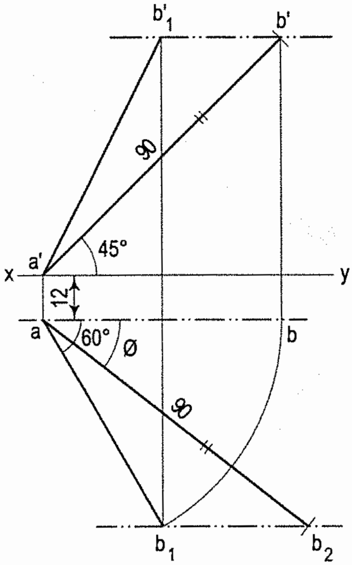


FIG. 10-39

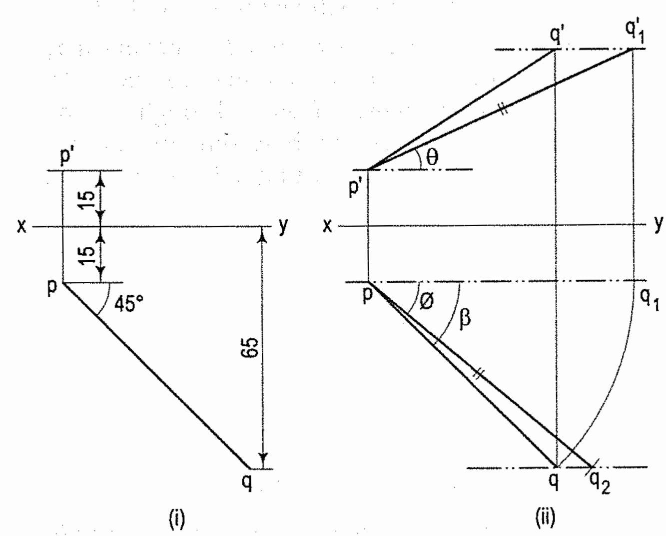


FIG. 10-40

Problem 10-15. (fig. 10-40): *Incomplete projections of a line PQ, inclined at 30° to the H.P. are given in fig. 10-40(i). Complete the projections and determine the true length of PQ and its inclination with the V.P.*

(i) Turn the top view *pq* [fig. 10-40(ii)] to a position pq_1, so that it is parallel to *xy*. Through *p'*, draw a line making an angle θ (equal to 30°) with *xy* and cutting the projector through q_1 at q'_1. $p'q'_1$ is the front view of *PQ*.

(ii) Through q'_1, draw a line parallel to *xy* and cutting the projector through *q* at *q'*. *p'q'* is the front view of *PQ*.

(iii) With *p* as centre and radius equal to $p'\ q'_1$, draw an arc cutting the locus of *q* at q_2.

(iv) Join *p* with q_2. ø is the inclination of *PQ* with the V.P.

Problem 10-16. (fig. 10-41): *The end A of a line AB is 25 mm behind the V.P. and is below the H.P. The end B is 12 mm in front of the V.P. and is above the H.P. The distance between the projectors is 65 mm. The line is inclined at 40° to the H.P. and its H.T. is 20 mm behind the V.P. Draw the projections of the line and determine its true length and the V.T.*

Draw the top view *ab* and mark the H.T. on it, 20 mm above *xy*.

We have seen that the line representing the true length obtained by the trapezoid method, intersects the top view or the top view-produced, at the H.T. at an angle equal to the true inclination of the line with the V.P.

(i) Hence, at the ends *a* and *b*, draw perpendiculars to *ab* on its opposite sides (as one end is below the H.P. and the other end above it). Through the H.T., draw a line making angle θ (equal to 40°) with *ab* and cutting the perpendiculars at A_1 and B_1, as shown. A_1B_1 is the true length of *AB*. aA_1 and bB_1 are the distances of the ends *A* and *B* respectively from the H.P.

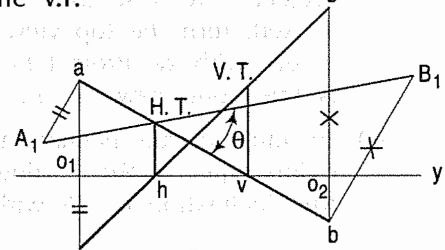


FIG. 10-41

(ii) Project *a* and *b* to *a'* and *b'*, making $a'o_1$ equal to aA_1 and $b'o_2$ equal to bB_1. *a'b'* is the front view of *AB*. Through *v*, the point of intersection between *ab* and *xy*, draw a projector cutting *a'b'* at the V.T. of the line.

Problem 10-17. (fig. 10-42): *A line AB, 90 mm long, is inclined at 30° to the H.P. Its end A is 12 mm above the H.P. and 20 mm in front of the V.P. Its front view measures 65 mm. Draw the top view of AB and determine its inclination with the V.P.*

(i) Mark *a* and *a'* the projections of the end *A*. Through *a'*, draw a line *ab'* 90 mm long and making an angle of 30° with *xy*.

(ii) With *a'* as centre and radius equal to 65 mm, draw an arc cutting the path of *b'* at b'_1. $a'b'_1$ is the front view of *AB*.

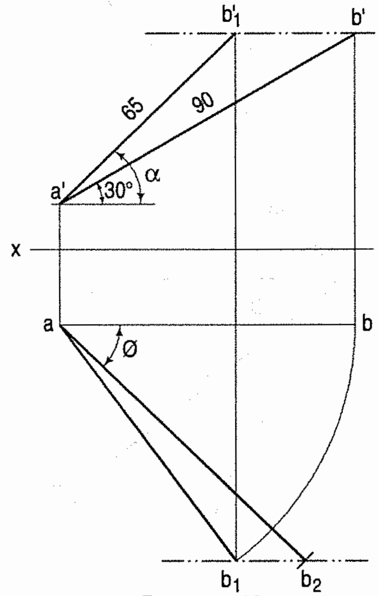


FIG. 10-42

(iii) Project *b'* to *b*, so that *ab* is parallel to *xy*. *ab* is the length of *AB* in the top view.

(iv) With *a* as centre and radius equal to *ab*, draw an arc cutting the projector through b'_1 at b_1. Join *a* with b_1. ab_1 is the required top view.

Determine ø as described in problem 10-14.

Problem 10-18. (fig. 10-43): *The ends of a line PQ are on the same projector. The end P is 30 mm below the H.P. and 12 mm behind the V.P. The end Q is 55 mm above the H.P. and 45 mm in front of the V.P. Determine the true length and traces of PQ and its inclinations with the two planes.*

Note: When the ends of a line are on the same projector or sum of angles of inclinations of a line with *xy* is 90°, use Method II only.

Draw the projections *pq* and *p' q'* as per given positions of the ends *P* and *Q*. They will partly coincide with each other.

(i) At the ends *p* and *q* of the top view *pq*, erect perpendiculars, viz. pP_1 equal to *p'o*, and qQ_1 equal to *q'o* and on opposite sides of *pq*. P_1Q_1 is the true length of *PQ*. θ is the inclination of *PQ* with the H.P. and the point of intersection between P_1Q_1 and *pq* is the H.T. of *PQ*.

(ii) Similarly, draw perpendiculars to $p'q'$. viz. $p'P_2$ equal to po and $q'Q_2$ equal to qo and on opposite sides of $p'q'$. P_2Q_2 is the true length. ø is the true inclination of PQ with the V.P. and the point where P_2Q_2 cuts $p'q'$ is the V.T. of PQ.

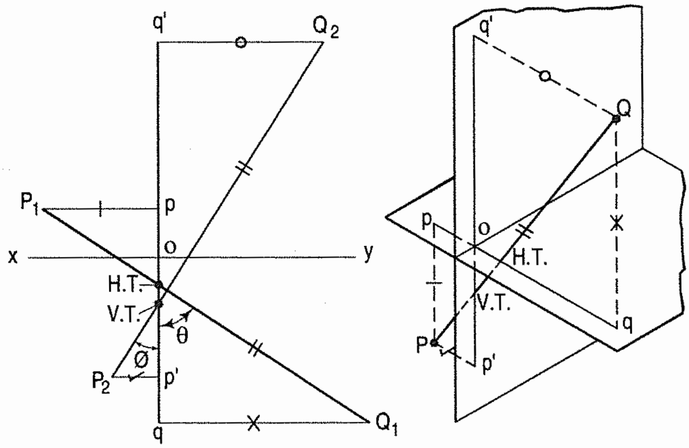


FIG. 10-43

Problem 10-19. (fig. 10-44): *A line AB, inclined at 40° to the V.P., has its ends 50 mm and 20 mm above the H.P. The length of its front view is 65 mm and its V.T. is 10 mm above the H.P. Determine the true length of AB, its inclination with the H.P. and its H.T.*

(i) Draw the front view $a'b'$ as per given positions of A and B and the given length.

(ii) Draw a line parallel to and 10 mm above xy. This line will contain the V.T. Produce $a'b'$ to cut this line at the V.T. Draw a projector through V.T. to v on xy.

(iii) Assuming a' V.T. to be the front view of a line which makes 40° angle with the V.P. and whose one end v is in the V.P., let us determine its true length.

(iv) Keeping V.T. fixed, turn the end a' to a'_1 so that the line becomes parallel to xy. Through v, draw a line making an angle of 40° with xy and cutting the projector through a'_1 at a_1. The line through a_1, drawn parallel to xy, is the locus of A in the top view. Project a' to a on this line. av is the top view of the line, whose front view is a' V.T. and whose true length is equal to a_1v.

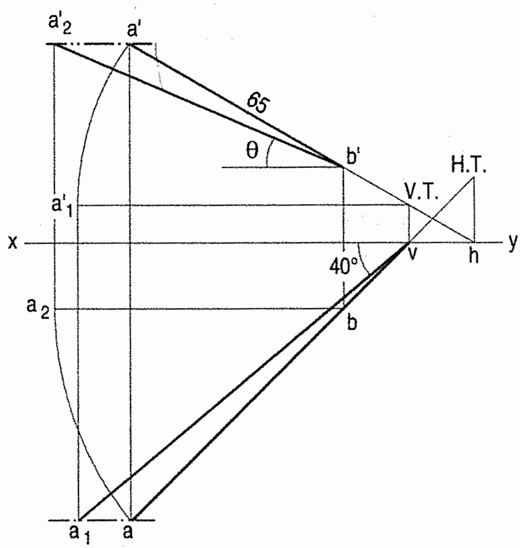


FIG. 10-44

(v) But $a'b'$ is the given front view of AB. Therefore, project b' to b on av. ab is the top view of AB. Obtain the inclination θ with the H.P. by making the top view ab parallel to xy, as shown.

Produce *a'b'* to meet *xy* at *h*. Draw a projector through *h* to cut *ab*-produced, at the H.T. of the line.

Problem 10-20. (fig. 10-45): *The front view a'b' and the H.T. of a line AB, inclined at 23° to the H.P. are given in fig. 10-45(i). Determine the true length of AB, its inclination with the V.P. and its V.T.*

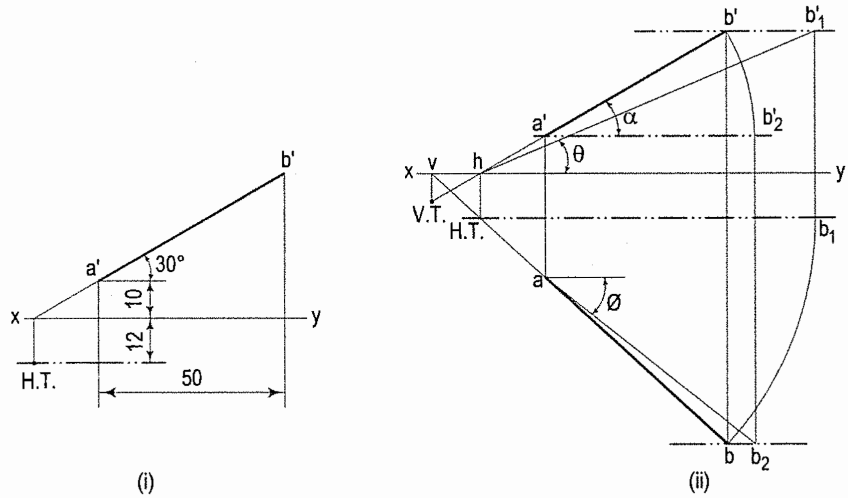


FIG. 10-45

Consider that *hb'* is the front view of a line inclined at 23° to the H.P. and the top view of whose one end is in H.T.

(i) Through *h* [fig. 10-45(ii)], draw a line making an angle of $\theta = 23°$ with *xy* and cutting the locus of *B* in the front view in b'_1. hb'_1 is the true length of the line whose length in the top view is H.T. b_1.

(ii) With H.T. as centre and radius equal to H.T. b_1, draw an arc cutting the projector through *b'* at *b*. H.T. *b* is the top view and *hb'* is the front view of a line which contains *AB*.

(iii) Therefore, through a', draw a projector cutting H.T. *b* at a. *ab* is the top view of *AB*.

Obtain the true length ab_2 (of *AB*) and its inclination ø with the V.P. by making *a' b'* parallel to *xy*.

(iv) Produce *ba* to meet *xy* in *v*. Draw a projector through *v* to cut *b'a'*-produced, at the V.T. of the line.

Problem 10-21. (fig. 10-46): *A tripod stand rests on the floor. One of its legs is 150 mm long and makes an angle of 70° with the floor. The other two legs are 163 mm and 175 mm long respectively. The upper ends of the legs are attached to the corners of a horizontal equilateral triangular frame of 50 mm side, one side of which is parallel to the V.P. In the top view, the legs appear as lines 120° apart, which if produced, would meet in a point. Draw the projections of the tripod and determine the angle which each of the other two legs makes with the floor. Assume the thickness of the frame and of the legs to be equal to that of the line.*

(i) At any point *P* on *xy*, draw a line *PA*, 150 mm long and making 70° angle with *xy*. *h* is the height of the tripod and PA_1 is the length of the leg in the top view.

(ii) Draw an equilateral triangle *abc* of 50 mm side with one side parallel to and below *xy*. Project the front view *a'b'c'* at the height *h* above *xy*. Determine the lengths of the other two legs in the top view as described below.

(iii) With *b'* as centre and radius equal to 163 mm, draw an arc cutting *xy* in q'_1. Similarly with *a'* and *c'* as centre and radius equal to 150 mm and 175 mm, draw an arc cutting *xy* in p'_1 and r'_1 respectively ap_1, bq_1 and cr_1 are the lengths of the three legs in the top view and α and β respectively are their inclinations with the floor (H.P.).

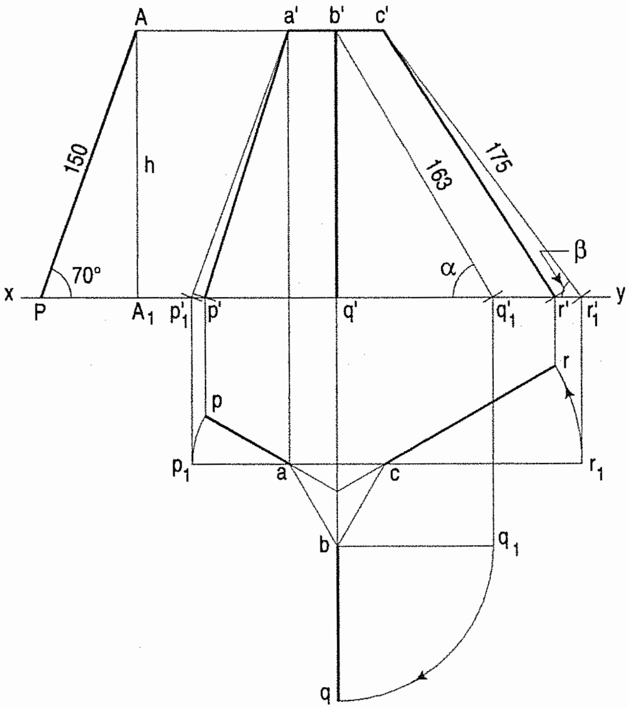


FIG. 10-46

(iv) The legs in the top view are to be inclined at 120° to each other and to meet at a point, if produced. Therefore, draw lines bisecting the angles of the triangle, making *ap* equal to PA_1, *bq* equal to bq_1 and *cr* equal to cr_1, thus completing the top view.

(v) Project *p,q* and *r* to *p',q'* and *r'* respectively on *xy*. Complete the front view by drawing lines *a'p'*, *b'q'* and *c'r'*.

Problem 10-22. (fig. 10-47): *A straight road going uphill from a point A, due east to another point B, is 4 km long and has a slope of 15°. Another straight road from B, due 30° east of north, to a point C is also 4 km long but is on ground level. Determine the length and slope of the straight road joining the points A and C. Scale, 10 mm = 0.4 km.*

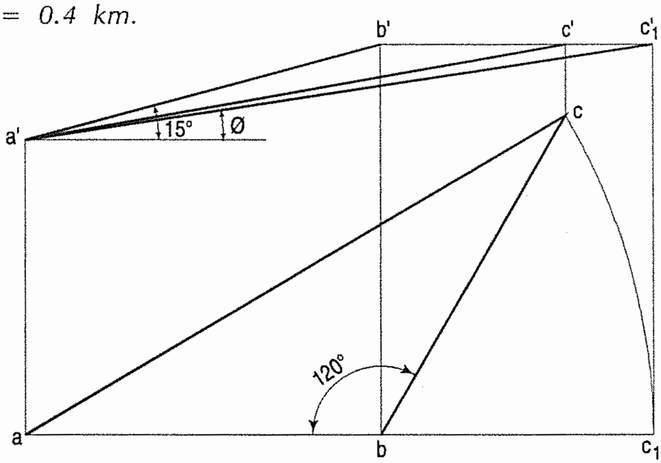


FIG. 10-47

(i) Mark any point a'. Draw a line $a'b'$, 100 mm long, to the right of a' and inclined upwards at 15° to the horizontal (to represent the road from *A* to *B*). Project its top view *ab* keeping it horizontal.

(ii) As the road from *B* to *C* is on ground level, the top view *bc* will be equal to 100 mm and inclined at (90° + 30°) i.e. 120° to *ab*.

(iii) From *b*, draw a line *bc*, 100 mm long and making 120° angle with *ab*. Project *c* to c' making $b'c'$ horizontal. $a'c'$ and *ac* are the front view and the top view respectively of the road from *A* to *C*,

Determine the true length $a'c'_1$ and the angle ø as shown, which are respectively the length and slope of the road from *A* to *C*.

Problem 10-23. (fig. 10-48): *Two lines AB and AC make an angle of 120° between them in their front view and top view. AB is parallel to both the H.P. and the V.P. Determine the real angle between AB and AC.*

Draw any line $b'a'$ parallel to and above *xy*, and another line $a'c'$ of any length making 120° angle with $b'a'$. Join b' with c'.

(i) Project the top view *ba* parallel to *xy* and the top view *ac*, making 120° angle with *ba*. Join *b* with *c*. $b'a'$ or *ba* is the true length of *AB*. Determine the true lengths of *AC* and *BC*, viz. $a'c'_1$ and $b'c'_2$, as shown.

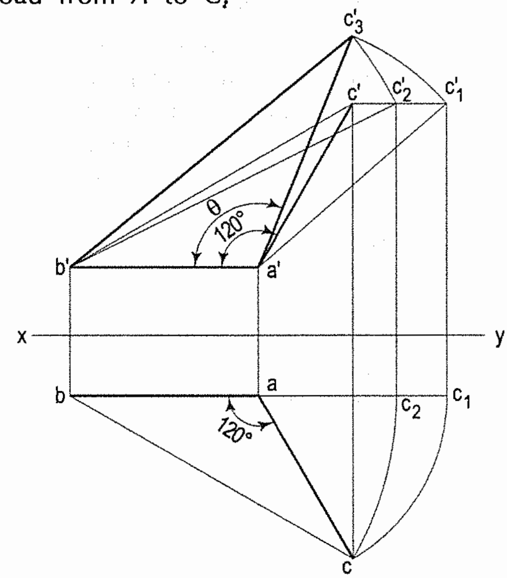


FIG. 10-48

(ii) Draw a triangle $a'b'c'_3$ making $a'c'_3$ equal to $a'c'_1$ and $b'c'_3$ equal to $b'c'_2$. $\angle b'a'c'_3$ is the real angle between *AB* and *AC*.

Problem 10-24. (fig. 10-49): *An object O is placed 1.2 m above the ground and in the centre of a room 4.2 m × 3.6 m × 3.6 m high. Determine graphically its distance from one of the corners between the roof and two adjacent walls. Scale, 10 mm = 0.5 m.*

(i) Draw the front view (of the room) $a'b'c'd'$ as seen from the front of, say 3.6 m wall. $a'b'$ is the width of the room and $a'd'$ is the height. The front view o' of the object will be seen 1.2 m above the mid-point of $a'b'$. c' and d' are the top corners of the room. $o'c'$ is the front view of the line joining the object with a top corner.

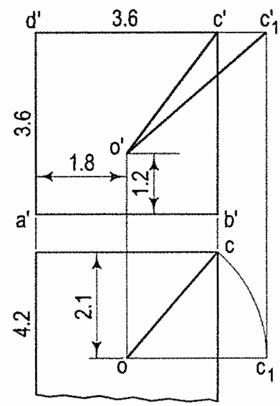


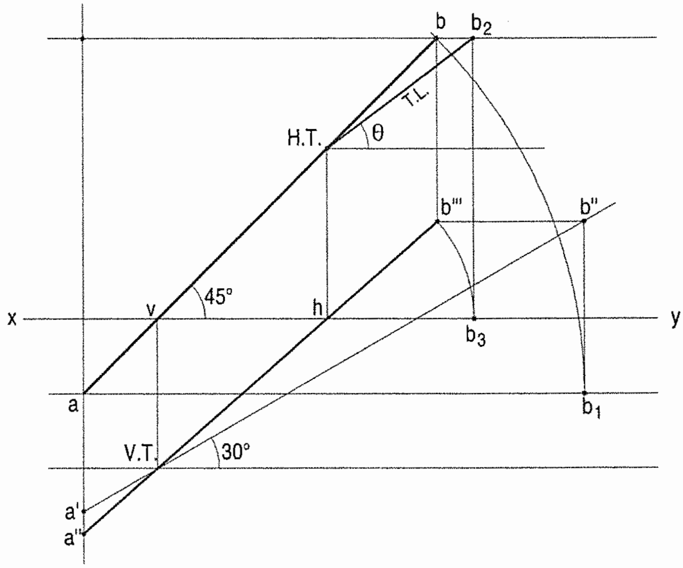
FIG. 10-49

(ii) Draw the top view of the room. It will be a rectangle of sides equal to 3.6 m and 4.2 m. The top view *o* of the object will be in the centre of the rectangle. *oc* is the top view of the line joining the object with the top corner.

Determine the true length $o'c'_1$, which will show the distance of the object from one of the top corners of the room.

Problem 10-25. *The straight line AB is inclined at 30° to H.P., while its top view at 45° to a line xy. The end A is 20 mm in front of the V.P. and it is below the H.P. The end B is 75 mm behind the V.P. and it is above the H.P. Draw the projections of the line when its V.T. is 40 mm below. Find the true length of the portion of the straight line which is in the second quadrant and locate its H.T.*

Refer to fig. 10-50.



T.L. = 50 mm. ∠V.P., θ = 37°

FIG. 10-50

(i) Mark the points *a* (top view of *A*) and *b* (top view of *B*) at the distances of 20 mm and 75 mm below and above *xy* respectively.

(ii) Through the point *a*, draw a line at 45° intersecting *xy* and the path of *b* at *v* and *b* respectively as shown.

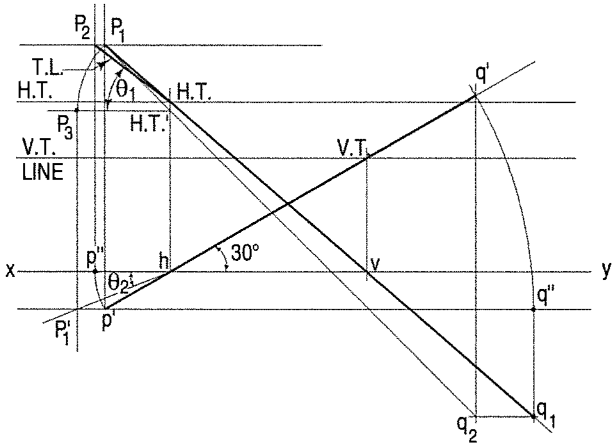
(iii) Construct a line containing V.T. 40 mm below *xy*. Draw perpendicular from *v* to the line V.T.

(iv) With *a* as centre and radius equal to *ab*, draw an arc which intersects at b_1 a line drawn from *a* parallel to *xy*. From V.T. draw a line at 30° intersecting the projector of b_1 at *b*''. From *b*'', draw *b*''*b*''' parallel to *xy* to intersect projector of *b* at *b*'''. Join V.T. *b*'''. Produce it to meet the projector from *a* at *a*''. *a*'' *b*''' is the required projection. *a*''*b*''' intersects line *xy* at *h*. From *h* draw the perpendicular to meet *ab*. The intersection point represents H.T.

(v) With *h* as centre and radius equal to *hb*''', draw an arc intersecting at b_3. Draw projector from b_3 to cut the path of *b* at b_2. Join H.T. b_2. Measure the angle H.T. b_2 with *xy*. This is an angle made by the line with the V.P.

Problem 10-26. (fig. 10-51.): *The front view of a line PQ makes an angle of 30° with xy. The H.T. of the line is 45 mm behind the V.P. While its V.T. is 30 mm above the H.P. The end P of the line is 10 mm below the H.P. and the end Q is in the first quadrant. The line is 150 mm long. Draw the projections of the line and determine the true-length of the portion of the line which is in the second quadrant. Also find the angle of the line with the H.P. and V.P.*

(i) Draw lines containing H.T. and V.T. at 45 mm and 30 mm above *xy* respectively. Mark point *p*' at 10 mm below line *xy*. Draw *p*'*q*' at 30° from *p*' intersecting *xy* at *h* and line V.T. at V.T. From *h*, draw perpendicular to line H.T. to locate H.T.



T.L. = 25 mm. ∠V.P., θ_1 = 37° ∠V.P., θ_2 = 22°

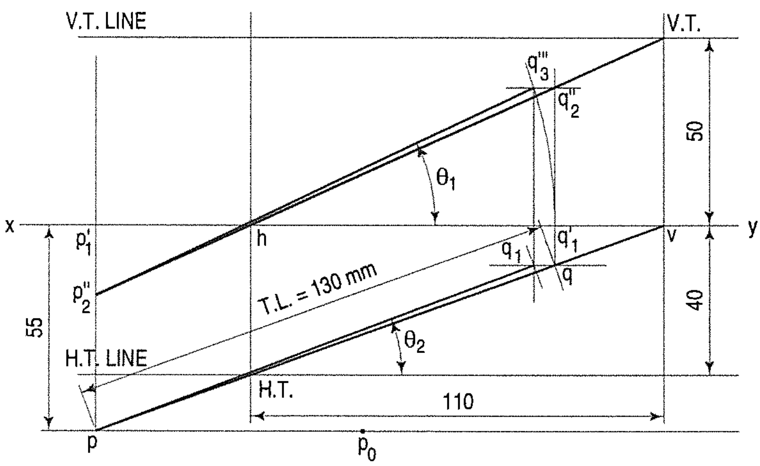
FIG. 10-51

(ii) Draw perpendicular from V.T. to intersect *xy* at *v*. Join *v* H.T. and produce to intersect the projector of *p*' at p_1. Draw $p_1 q_1$ of 150 mm representing true length of the line in top view. From *p*', draw line parallel to *xy* representing front view of the line *PQ*. Draw projector from q_1 to cut the line drawn from *p*' at *q*''.

(iii) Keeping *p*' fixed, turn *p*'*q*'' such that it cuts the line drawn from *p*' at *q*'. From q_1 draw line parallel to *xy* which intersects the vertical projector drawn from *q*' at q_2. Join P_1q_2. This is the required projection.

(iv) Keeping *h* fixed, rotate *hp*' and make it parallel to *xy*. From *P*'' draw projector intersecting the path of p_1 at p_2. H.T. p_2 is true-length of the line. Similarly keeping H.T.' fixed, turn H.T.' p_1 to make it parallel to *xy* as shown. From P_3, draw projector to intersect the horizontal line drawn from $P'P'_1$. Measure angle xhp'_1. This is the required angle with H.P.

Problem 10-27. (fig. 10-52): *The end P of a line PQ 130 mm long, is 55 mm in front of the V.P. The H.T. of the line is 40 mm in front of the V.P. and the V.T. is 50 mm above the H.P. The distance between H.T. and V.T. is 110 mm. Draw the projections of the line PQ and determine its angles with the H.P. and the V.P.*



∠H.P., θ_1 = 25.5°; ∠V.P., θ_2 = 21°

FIG. 10-52

(i) Mark p below xy at a distance of 55 mm. Draw lines containing H.T. and V.T. at 40 mm below and 50 mm above xy respectively. Construct projectors through H.T. and V.T. 110 mm apart intersecting xy at h and v respectively.

(ii) Draw perpendiculars from h and v intersecting the lines containing H.T. and V.T. Join H.T. v and produce to cut the line drawn from point P as shown. Join h V.T. and extend further to intersect the projector drawn from P at P''_2.

(iii) Mark true length 130 mm on H.T.v. Let it be pq. Draw projector from q cutting xy at q'_1. With p'_1 as centre and radius equal to $p'_1q'_1$, draw an arc cutting a horizontal line drawn from q''_2 at q'''_3. Join pq_1 and $p''_2q'''_3$, are the required projections. $\theta_1 = 25.5°$ and $\theta_2 = 21°$ are the measured angles.

Problem 10-28. (fig. 10-53): *The distance between the end projectors of a line AB is 70 mm and the projectors through the traces are 110 mm apart. The end of a line is 10 mm above H.P. If the top view and the front view of the line make 30° and 60° with xy line respectively, draw the projections of the line and determine* (i) *the traces,* (ii) *the angles with the H.P. and the V.P.,* (iii) *the true length of the line. Assume that the line is in the first quadrant.*

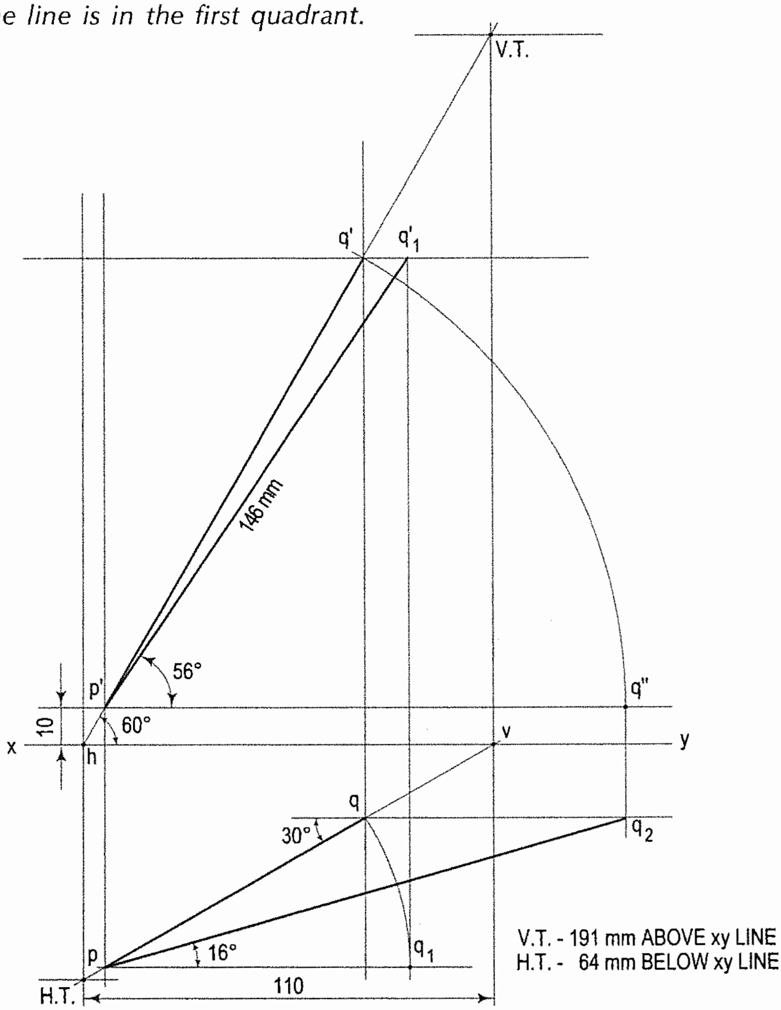


FIG. 10-53

(i) Draw two vertical lines 110 mm apart representing projectors through the traces of the line. Mark intersection of these projectors *h* and *v* on *xy* as shown.

(ii) From *v* and *h*, draw lines at 30° and 60°.

(iii) Mark *p*', 10 mm above *xy*. Draw two vertical projectors at 70 mm apart keeping equal distance from the projectors through traces. *p'q'* and *pq* represent front view and top view of the line as shown.

(iv) Keeping *p* fixed, turn *pq* to position pq_1. From q_1, draw a vertical projector intersecting the path of *q*' at q'_1. Join $p'q'_1$. This is the true length of the line. Measure angle $q''p'q'_1$ with the horizontal line as shown. This is the angle made by the line with the H.P. Similarly rotate *p'q'* making it parallel to *xy* as shown.

Draw a vertical projector from *q*" to intersect the path of *q* at q_2. Measure the angle q_1pq_2 with the horizontal line. This is the angle made by the line with V.P.

Note: Problem 10-29 and problem 10-31 are solved by using auxiliary plane method.

Problem 10-29. *Two pipes PQ and RS seem to intersect at a' and a in front view and top view as shown in fig. 10-54. The point A is 400 mm above H.P. and 300 mm in front of a wall.*

Neglecting the thickness of the pipes, determine the clearance between the pipes.

Refer to fig. 10-54.

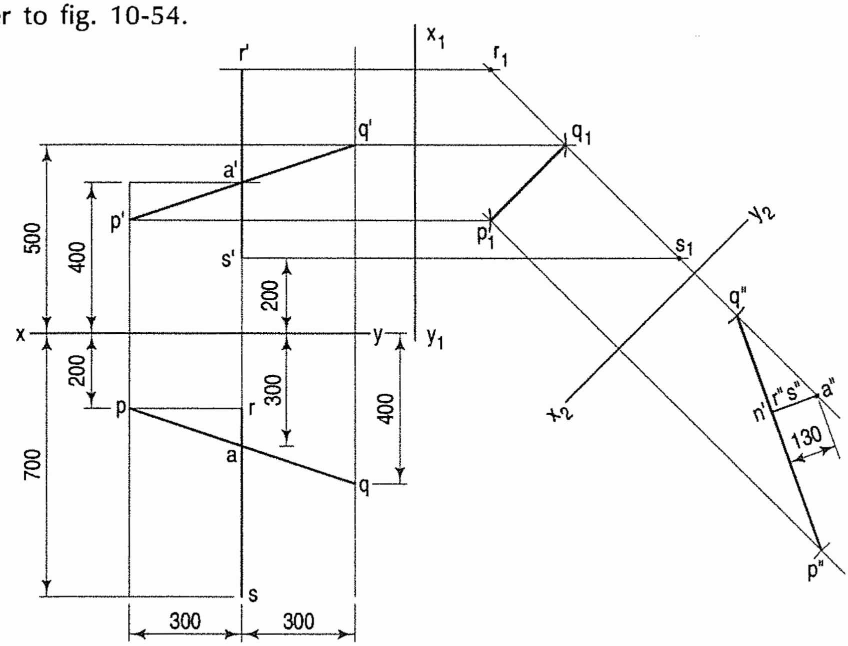


FIG. 10-54

(i) Draw the projections of pipes treating them as lines.

(ii) Draw x_1y_1 perpendicular to the line *xy* and project auxiliary top view. Note that the distances of p_1, q_1, r_1 and s_1 from x_1y_1 are equal to the distances of *p*, *q*, *r* and *s* from *xy*.

(iii) Mark an auxiliary reference line x_2y_2 perpendicular to the line r_1s_1 for obtaining the point view of the line. Draw an auxiliary front view as shown. Note that the distances of p'', q'', r'' and s'' from x_2y_2 are equal to the distances of p', q', r' and s' from x_1y_1.

(iv) a'' n' represents clearance between two pipes which is approximately 130 mm.

Problem 10-30. *The end projectors of line AB are 22 mm apart. A is 12 mm in front of the V.P. and 12 mm above the H.P. The point B 6 mm in front of the V.P. and 40 mm above the H.P. Locate the H.T. and the V.T. of the line and also determine its inclinations with the V.P. and the H.P.*

If the line *AB* is shifted to II, III and IV quadrants as shown in fig. 10-55 (assume that the distances of *A* and *B* from the projection-planes are same as the first quadrant), draw the projections of line and locate the traces.

The solution of first part of problem is shown in pictorial view. For the second part of the problem, the locations of the line in the respective quadrants are shown in fig. 10-55.

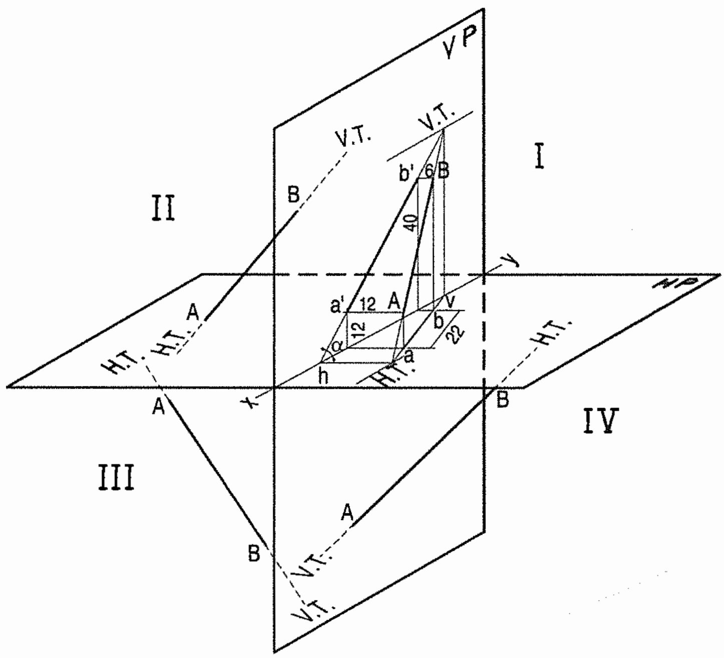


FIG. 10-55

Students are advised to draw the orthographic projections of the line for the respective quadrants.

Problem 10-31. *The end projectors of line PQ are 80 mm apart. The end P is 15 mm in front of the V.P. and 60 mm above the H.P. While Q is 50 mm in front of the V.P. and 10 mm above the H.P. A point R is situated on the projector at a distance of 31 mm from the projector through P measured towards the projector of Q. The point R is 70 mm in front of the V.P. and above the H.P. A perpendicular is drawn from R on PQ. Draw its projections.*

Refer to fig. 10-56.

(i) Draw two end projectors of the line *PQ* at 80 mm apart.

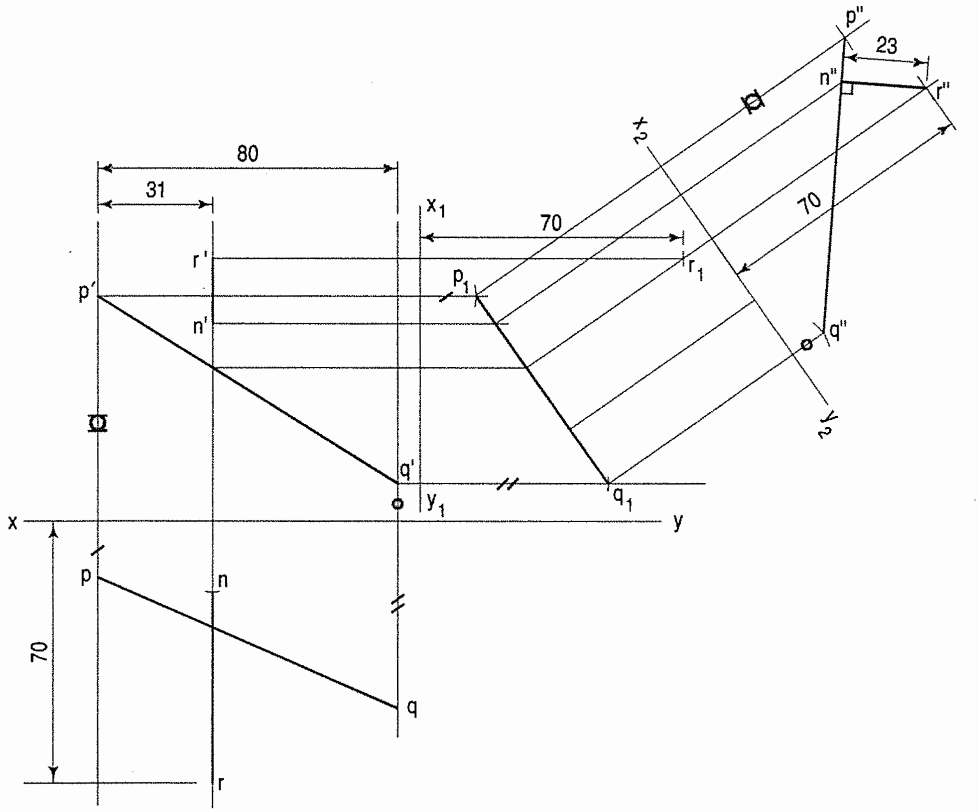
(ii) Mark the front view *p'* and the top view *p* at 60 mm and 15 mm from *xy* on the end projector of *P*. Similarly mark *q'* and *q* at 10 mm and 50 mm from *xy* on the end projector of *Q*.

(iii) Draw a vertical line at 31 mm away from *p'p* towards *q'q*. Mark the position of *R* in the top view *r* and the front view *r'* at 70 mm from *xy* as shown.

(iv) Draw x_1y_1 perpendicular to *xy* as shown. Draw projectors from *p'*, *q'* and *r'* on x_1y_1. Transfer the distances 15 mm, 50 mm and 70 mm of *p*, *q* and *r* from *xy* to the new top view p_1, q_1 and r_1 from x_1y_1.

(v) Draw another reference line x_2y_2 for the new front view parallel to p_1q_1. Transfer the distances 60 mm, 10 mm and 70 mm of *p'*, *q'* and *r'* from *xy* to the new front view *p"*, *q"* and *r"* from x_2y_2.

(vi) From *r"*, draw perpendicular to *p"q"* intersecting at *n"* as shown which is measured as 23 mm.



n"r" = 23 mm

FIG. 10-56

Problem 10-32. *The end projectors of a line PQ are 65 mm apart. P is 25 mm behind the V.P. and 30 mm below the H.P. The point Q is 40 mm above the H.P. and 15 mm in front of the V.P. Find the third point C in the H.P. and in front of the V.P. such that its distance from a point P is 45 mm and that from Q is 60 mm. Determine inclinations of PQ with the H.P. and the V.P.*

Refer to fig. 10-57.

(i) Draw the end projectors of the line *PQ* 65 mm apart. Mark the projection of ends *P* and *Q* according to given distances. *p'q'* and *pq* are the front view and the top view respectively.

(ii) Mark the front view of point *c* in the line *xy* because it is lying in the H.P. Extend the projection line from *c'* to *c* on the top view *pq*.

(iii) Keeping *c* fixed, turn *cp* to cp_2 making parallel to *xy*.

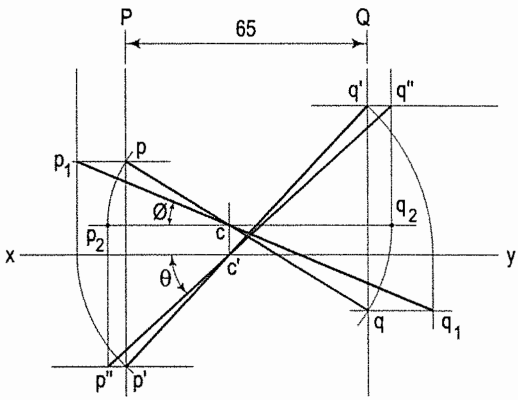


FIG. 10-57

(iv) Project p_2 to *p"*. Join *c'p"*. This is the true distance of the line *cp*. Similarly turn *cq* to cq_2 making it parallel to *xy*. Project q_2 to *q"* join *c'q"*. This represents the true distance of the line *cq*.

(v) Measure angles θ and ø as shown.

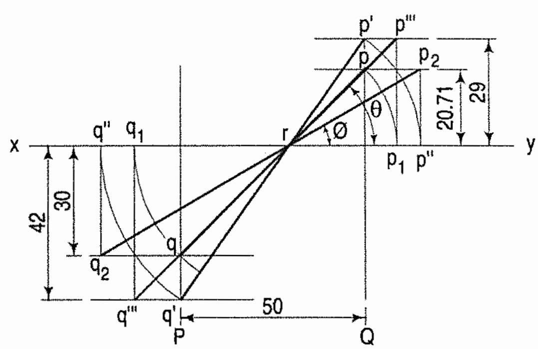
Problem 10-33. (fig. 10-58): *The distance between the end-projectors of a line PQ is 50 mm. A point P is 29 mm above H.P. and 20.71 mm behind V.P. While a point Q is 42 mm below H.P. and 30 mm in front of V.P.*

Draw the projections of the line and determine the true length and the true inclinations of line with H.P. and V.P.

(i) Draw the end-projectors 50 mm apart. Mark *p'* and *p*, the front view and the top view of the end *P* at 29 mm and 20.71 mm respectively. Similarly mark *q* and *q'* at 42 mm and 30 mm on the end-projector *Q* as shown. Join *p'q'* and *pq*. They are intersecting *xy* at *r*. Mark paths of *p'*, *q'*, *p* and *q* parallel to *xy*.

(ii) With centre *r* and radius *rp'*, draw an arc intersecting *xy* at *p"*. Through *p"*, draw projector cutting the path of *p* at p_2. Similarly with the same centre and radius *rq'*, draw an arc intersecting *xy* at *q"*. Through *q"*, draw projector cutting the path of *q* at q_2. Join p_2q_2 which represents true length. Measure ø angle made by p_2q_2 with *xy* at *r*. This is an angle made by the line with V.P.

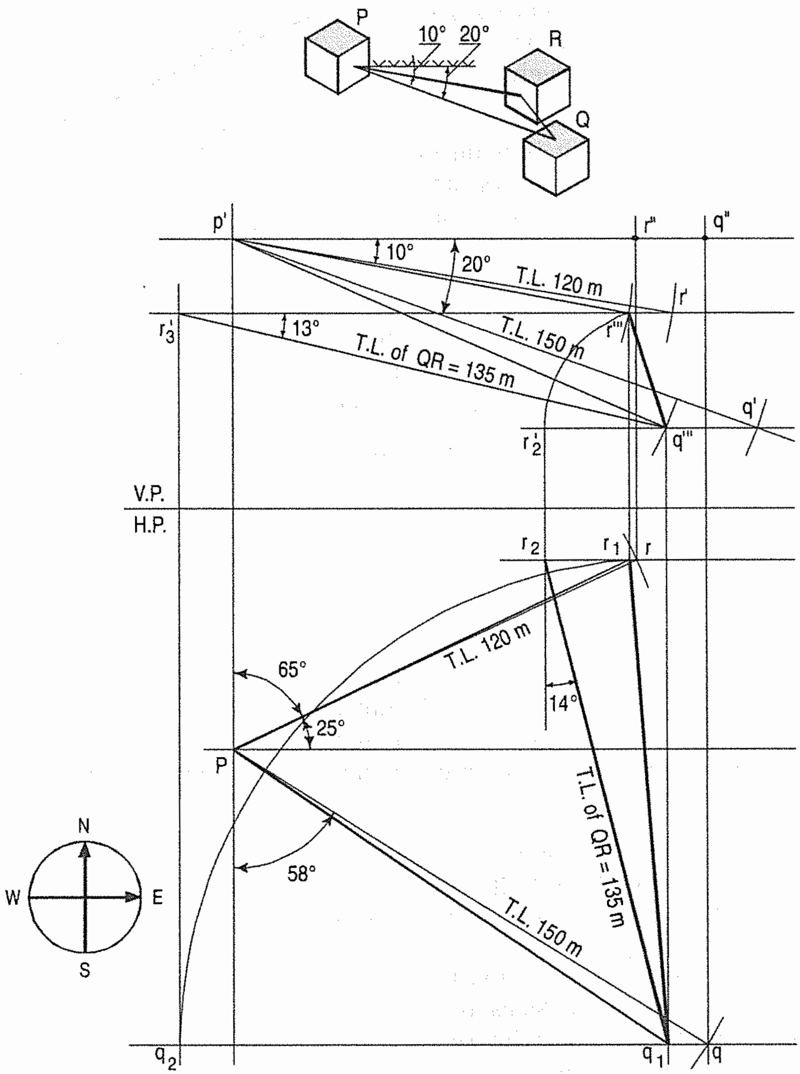
(iii) Similarly centre as *r* and radii *rp* and *rq*, draw arcs intersecting *xy* at p_1 and q_1 respectively. Through p_1 and q_1 draw projectors to intersect the paths of *p'* and *q'* at *p'''* and *q'''*. Join *p'''q'''*. Measure θ angle made by it with *xy*. This is an angle by line with H.P.



∠θ = 45°; ∠ø = 30°, true length = 100 mm

FIG. 10-58

Problem 10-34. (fig. 10-59): *Two pipes emerge from a common tank. The pipe PQ is 150 metre long and bears S 58° E on a downward slope of 20°. The pipe PR is 120 metre long and bears N 65° E on a downward slope of 10°. Determine the length of pipe required to connect Q and R. Take scale 1 mm = 1 m.*



T.L. of QR = 135 m. S 13 E at downward slope of 13°

FIG. 10-59

(i) Mark position of *P* in the top view and the front view as shown.

(ii) At *P*, draw a line *pq* 150 mm at 58° with the vertical measuring anti-clockwise as the angle is required to measure from south to east. Similarly draw a line *pr* 120 mm at 25° measured from east to north.

(iii) Draw the horizontal lines from *q* and *r*.

(iv) From point *p*', draw a line *p'q'* 150 mm and *p'r'* 120 mm at 20° and 10° with the horizontal lines. Draw horizontal lines from *r*' and *q*'.

(v) Draw a projector from q to intersect the horizontal line drawn from p' at q''. $p'q''$ is the front view of PQ. With p' as centre and radius equal to $p'q''$, draw an arc intersecting the path of q' at q'''. Join $p'q'''$ and draw a projector from q''' cutting the path of q at q_1. Then $p'q'''$ and pq_1 are required projection. Similarly obtain the projection of $p'r'''$ and pr_1 for the line PR as shown. Join $r'''q'''$ and r_1q_1. With centre q''' and a radius $q'''r'''$, draw an arc intersecting the path of q''' at r'_2. Draw a projector from r'_2 cutting the path of r at r_2. Join q_1r_2 and measure its true length and angle.

(vi) With centre q_1 and radius q_1r_1, draw an arc intersecting the path of q at q_2. Draw a projector from q_2 cutting the path of r' at r'_3. Join $q'''r'_3$ and measure its true length and angle.

(vii) QR = 135 m is the measured true length and S 13 E at downward slope of 13° is the measured angle.

Note: Depression or front view angles are seen in front view while bearing angles are seen in top view.

Problem 10-35. (fig. 10-60): *The projectors of two points P and Q are 70 mm apart. The point P is 25 mm behind the V.P. and 30 mm below the H.P. The point Q is 40 mm above the H.P. and 15 mm infront of the V.P. Find the third point S which is in the H.P. and infront of the V.P. such that its distance from point P is 90 mm and that from Q is 60 mm.*

(i) Draw xy line.

(ii) Draw the projectors of P and Q 70 mm apart.

(iii) Mark on the projector of the point p the front view and top view of the point p at 25 mm and 30 mm from xy respectively, say p' and p. Similarly on the projector of the point Q, mark the front view q' and the top view q for given distance from the xy.

(iv) Join $p'q'$ and pq. They are the front view and the top view of the line PQ.

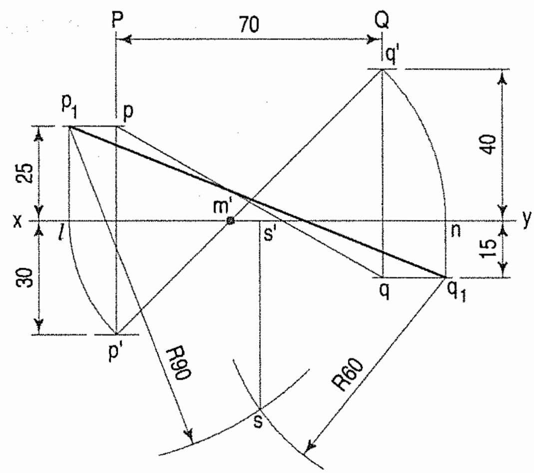


FIG. 10-60

(v) The front view $p'q'$ intersects xy line at m'. Taking m' as centre, $m'p'$ and $m'q'$ as radii rotate so that $p'q'$ becomes parallel to the xy line or intersect xy line at l and n.

(vi) Draw the projectors from l and n to intersect path of the point p and the point q at p_1 and q_1. Join p_1 and q_1, it represents true length of the line PQ.

(vii) Now p_1 as centre and 90 mm radius, draw the arc. Take q_1 as centre and 60 mm as radius, draw the another arc so that it intersects previous arc at s. From s draw the projector to intersect xy line at s', which is front view of s.

Problem 10-36. (fig. 10-61): *Two unequal lines PQ and PR meeting at P makes an angle of 130° between them in their front view and top view. Line PQ is parallel and 6 mm away from both the principal planes. Assume the front view length of PQ and PR 50 mm and 60 mm respectively. Determine real angle between them.*

(i) Mark reference line *xy*. Draw at convinent distance above and below two parallel lines to the *xy* representing the front view and top view of *PQ* as shown. At the point *P'* construct angle of 130° with the length of *p'q'* and *p'r'* of 50 mm and 60 mm respectively.

(ii) Draw projector from *r'* to intersect the line at angle of 130° at *p* in the top view of *pq*.

(iii) Make *pr* and *qr* parallel to *xy* line and project above *xy* to intersect the path of *r'* at *r''* and *r'''* respectively.

Join *p'r'''* and *q'r''*. They are true length of the line *PR* and the line *QR*.

(iv) Construct triangle with sides *PQ* = 50 mm, *PR* = *p'r'''* and *QR* = *q'r''* as shown. Measure angle ∠*QPR* equal and it is 120° approximately.

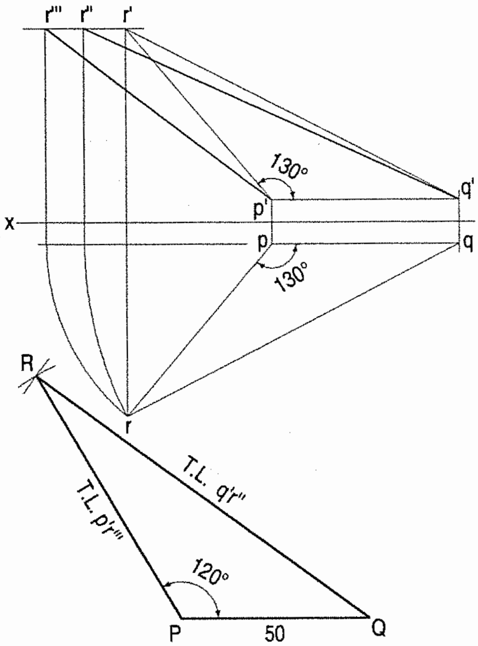


FIG. 10-61

Problem 10-37. (fig. 10-62): *The distance between end projectors of a straight line AB is 80 mm. The point A is 15 mm below the H.P. and 20 mm infront of the V.P. B is 60 mm behind the V.P. Draw projections of the line if it is inclined at 45° to V.P. Determine also true length and inclination with the H.P.*

(i) Draw *xy* line and two vertical parallel lines 80 mm apart showing the end projectors of the line *AB*.

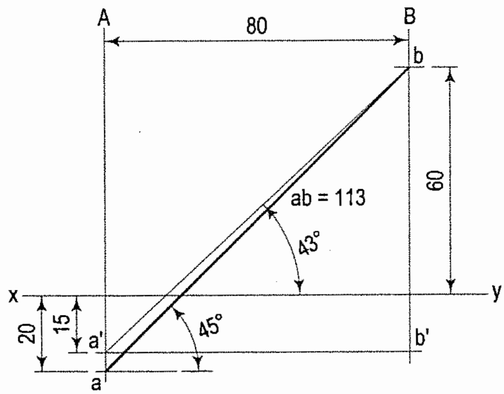


FIG. 10-62

(ii) Mark the point a' and the point a 15 mm and 20 mm below *xy* line on the projector of *A*.

(iii) Mark the point *b* above *xy* line at distance of 60 mm on the projector of *B*. Join the point *a* and the point *b*.

(iv) If we measure angle of *ab* with *xy* line it is 45° which is also inclination of the line *AB* with V.P.

(v) Therefore *ab* shows true length. From *a'* draw parallel line to *xy* to intersect the projector of *B* at *b'*. It is front view of the line *AB*. Measure angle of a'*b* with *xy* line. It is 43° with *xy* line.

Problem 10-38. (fig.10-63): *Two mangoes on a tree are respectively 2.0 m and 3.5 m above the ground and 1.5 m and 2.0 m away from 0.2 m thick compound wall, but on the opposite sides of it. The distance between the mangoes, measured along the ground and parallel to the wall is 2.7 m. Determine the real distance between the mangoes. Take scale 1 m = 10 mm.*

(i) Pictorial view is shown for understanding purpose.

(ii) Draw reference line (ground line) *xy*. Mark two parallel lines as end-projectors at 2.7 m (27 mm) apart.

(iii) Let *P* be mango behind the wall and *Q* be infront of the wall.

(iv) Mark *P*' and *P* along projector *P* to given distances. Similarly mark *q*' and *q* for given distances on the projector *Q*.

(v) Join the point *p* and the point *q*, the point *p*' and the point *q*'. Then *pq* and *p'q*' are projections of *PQ*.

(vi) Rotates *pq* taking *q* as centre, make it parallel to the ground line *xy*, intersecting at the poin P_1. Draw the projector from p_1. Draw line parallel to the ground line *xy* from *p*', intersecting at *p*".

(vii) Join *p"q*'. The line *p"q*' is true distance between mangoes *P* and Q. It is approximately 4.8 m.

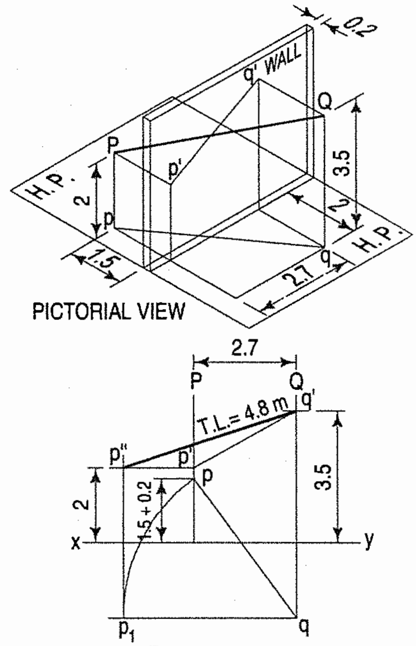


FIG. 10-63

Problem 10-39. (fig. 10-64): *The front view of straight line AB is 60 mm long and is inclined at 60° to the reference line xy. The end point A is 15 mm above H.P. and 20 mm infront of V.P. Draw the projections of a line AB if it is inclined at 45° to the V.P. and is situated in the first quadrant (Dihedral angle). Determine its true length, and inclination with the H.P.*

See fig. 10-64 which is self explanatory.

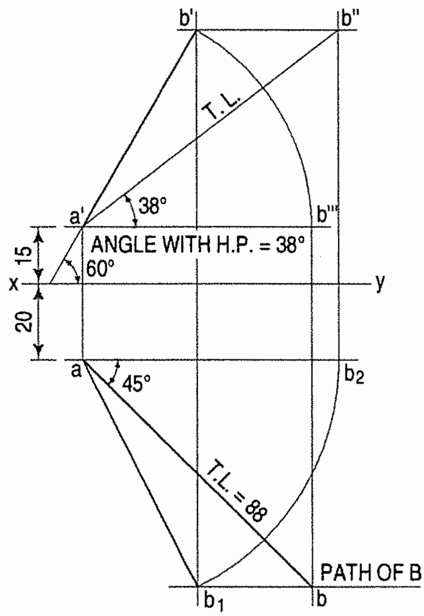


FIG. 10-64

Problem 10-40. (fig. 10-65): *A room 6 m × 5 m × 4 m high has a light-bracket above the centre of the longer wall and 1 m below the ceiling. The light bulb is 0.3 m away from the wall. The switch for the light is on an adjacent wall, 1.5 m above the floor and 1 m from the other longer wall. Determine graphically the shortest distance between the bulb and the switch.*

(i) Draw to scale 1 m = 10 mm front view and a top view of the room.

(ii) Mark mid point of longer wall (i.e. 6 m), say 6 mm.

(iii) Mark the point in the V.P. at distance 1 m (= 10 mm) from midpoint *m*. It is the front view of the light bulb, say *b*'.

(iv) From *xy* line at 0.3 m (i.e. 3 mm) on the projector passing through *b*' mark the point *b* (top view).

(v) Consider adjacent wall right side. Mark *s*' front view of the switch at 1.5 m (i.e. 15 mm) above *xy* line and on the same line mark *s* (top view of switch) at 4 m away from *xy*.

(vi) Join *b*'*s*' and *bs* which are the front view and the top view of the line.

(vii) *s* as centre and *bs* as radius, draw the arc to intersect a parallel line passing through *s* at b_1. From b_1, draw projector to intersect a parallel line to *xy* drawn from *b*' at *b*". Join *s*'*b*".

(viii) Line *s*'*b*" shows shortest distance between the switch and the bulb. Here it is approximately 5 m.

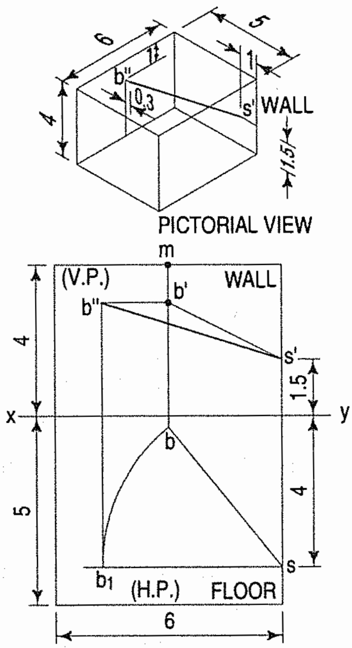
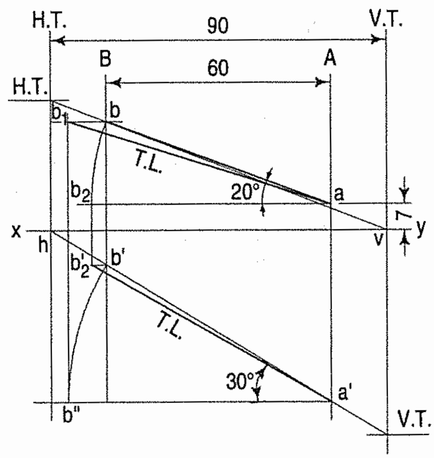


FIG. 10-65

Problem 10-41. (fig. 10-66): *The distance between end projectors of a line AB are 60 mm apart, while the projectors passing from H.T. and V.T. are 90 mm apart. The H.T. is 35 mm behind the V.P., the V.T. is 55 mm below the H.P. The point A is 7 mm behind the V.P. Find graphically true length of the line and inclinations with the H.P. and the V.P.*

See fig. 10-66 which is self-explanatory.



True length = 74;
Angle with H.P. = 30°;
Angle with V.P. = 20°
FIG. 10-66

Problem 10-42. (fig. 10-67): *A line CD is inclined at 30° to the H.P. and it is in the first quadrant. The end C is 15 mm above the H.P. while the end D is in the V.P. The mid point M of of the line is 40 mm above H.P. The distance between the end projectors of the line is 70 mm. Draw the projections of the line CD and the mid point M. Determine graphically the length of front view and top view and true length of the line. Also determine inclination of the line with the V.P.*

(i) Draw *xy* line.

(ii) Draw two projectors 70 mm, apart.

(iii) On the projector of *C*, mark *c*' at 15 mm above *xy* line.

(iv) Draw a line parallel to *xy* at 40 mm, to represent the path of mid-point *M*.

(v) From *c*' draw a 30° inclined line t cut the path of mid-point at *m*'. *c*'*m*' is half true length. With *m*' as centre and radius equal to *c*'*m*', draw an arc cutting the 30° inclined line at *d*'. *c*'*d*' is true length. From *d*', draw a line parallel to *xy*, to represent path of *D* in front view.

(vi) The path of *D* will intersect the projector of *D* at *d*". Join *c'd*". It is front view of *CD*.

(vii) With c' as centre and radius equal to *c'd*", draw an arc cutting a line drawn parallel to xy from c' at *d'''*. Project *d'''* to cut a line drawn with c as centre and *c'd'* (true length) as radius, at d_3. From d_3, draw a line parallel to *xy*, repressenting path of *D* in top view.

(viii) Project *d*" to cut path of *D* in top view at d_4. Join cd_4. It is top view of *CD*.

(ix) The results are shown in fig. 10-67.

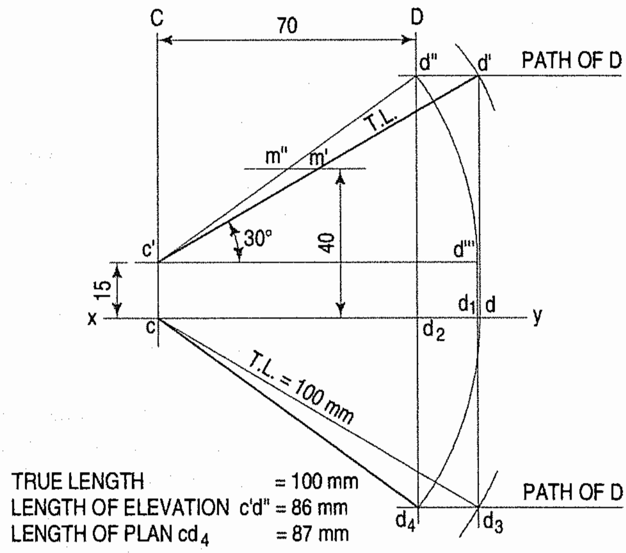


FIG. 10-67

EXERCISES 10(b)

1. A line *AB*, 75 mm long, is inclined at 45° to the H.P. and 30° to the V.P. Its end *B* is in the H.P. and 40 mm in front of the V.P. Draw its projections and determine its traces.
2. Draw the projections of a line *AB*, 90 mm long, its mid-point *M* being 50 mm above the H.P. and 40 mm in front of the V.P. The end *A* is 20 mm above the H.P. and 10 mm in front of the V.P. Show the traces and the inclinations of the line with the H.P. and the V.P.
3. The front view of a 125 mm long line *PQ* measures 75 mm and its top view measures 100 mm. Its end Q and the mid-point *M* are in the first quadrant, *M* being 20 mm from both the planes. Draw the projections of the line *PQ*.
4. A line *AB*, 75 mm long is in the second quadrant with the end *A* in the H.P. and the end *B* in the V.P. The line is inclined at 30° to the H.P. and at 45° to the V.P. Draw the projections of *AB* and determine its traces.
5. The end *A* of a line *AB* is in the H.P. and 25 mm behind the V.P. The end *B* is in the V.P. and 50 mm above the H.P. The distance between the end projectors is 75 mm. Draw the projections of *AB* and determine its true length, traces and inclinations with the two planes.
6. The top view of a 75 mm long line *CD* measures 50 mm. *C* is 50 mm in front of the V.P. and 15 mm below the H.P. *D* is 15 mm in front of the V.P. and is above the H.P. Draw the front view of *CD* and find its inclinations with the H.P. and the V.P. Show also its traces.
7. A line *PQ*, 100 mm long, is inclined at 45° to the H.P. and at 30° to the V.P. Its end *P* is in the second quadrant and Q is in the fourth quadrant. A point *R* on *PQ*, 40 mm from *P* is in both the planes. Draw the projections of *PQ*.
8. A line *AB*, 65 mm long, has its end *A* in the H.P. and 15 mm in front of the V.P. The end *B* is in the third quadrant. The line is inclined at 30° to the H.P. and at 60° to the V.P. Draw its projections.

9. The front view of a line *AB* measures 65 mm and makes an angle of 45° with *xy*. *A* is in the H.P. and the V.T. of the line is 15 mm below the H.P. The line is inclined at 30° to the V.P. Draw the projections of *AB* and find its true length and inclination with the H.P. Also locate its H.T.
10. A room is 4.8 m × 4.2 m × 3.6 m high. Determine graphically the distance between a top corner and the bottom corner diagonally opposite to it.
11. A line *AB* is in the first quadrant. Its end *A* and *B* are 20 mm and 60 mm in front of the V.P. respectively. The distance between the end projectors is 75 mm. The line is inclined at 30° to the H.P. and its H.T. is 10 mm above *xy*. Draw the projections of *AB* and determine its true length and the V.T.
12. Two oranges on a tree are respectively 1.8 m and 3 m above the ground, and 1.2 m and 2.1 m from a 0.3 m thick wall, but on the opposite sides of it. The distance between the oranges, measured along the ground and parallel to the wall is 2.7 m. Determine the real distance between the oranges.
13. Draw an isosceles triangle *abc* of base *ab* 40 mm and altitude 75 mm with *a* in *xy* and *ab* inclined at 45° to *xy*. The figure is the top view of a triangle whose corners *A*, *B* and *C* are respectively 75 mm, 25 mm and 50 mm above the H.P. Determine the true shape of the triangle and the inclination of the side *AB* with the two planes.
14. Three points *A*, *B* and *C* are 7.5 m above the ground level, on the ground level and 9 m below the ground level respectively. They are connected by roads with each other and are seen at angles of depression of 10°, 15° and 30° respectively from a point *O* on a hill 30 m above the ground level. A is due north-east, *B* is due north and *C* is due south-east of *O*. Find the lengths of the connecting roads.
15. A pipe-line from a point *A*, running due north-east has a downward gradient of 1 in 5. Another point *B* is 12 m away from and due east of *A* and on the same level. Find the length and slope of a pipe-line from *B* which runs due 15° east of north and meets the pipe-line from *A*.
16. The guy ropes of two poles 12 m apart, are attached to a point 15 m above the ground on the corner of a building. The points of attachment on the poles are 7.5 m and 4.5 m above the ground and the ropes make 45° and 30° respectively with the ground. Draw the projections and find the distances of the poles from the building and the lengths of the guy ropes.
17. A plate chimney, 18 m high 0.9 m diameter is supported by two sets of three guy wires each, as shown in fig. 10-68.

    One set is attached at 3 m from the top and anchored 6 m above the ground level. The other set is fixed to the chimney at its mid-height and anchored on the ground. Determine the length and slope with the ground, of one of the wires from each set.

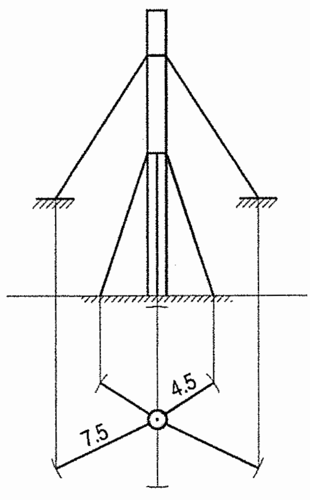


FIG. 10-68

18. The projectors drawn from the H.T. and the V.T. of a straight line *AB* are 80 mm apart while those drawn from its ends are 50 mm apart. The H.T. is 35 mm in front of the V.P., the V.T. is 55 mm above the H.P. and the end *A* is 10 mm above the H.P. Draw the projections of *AB* and determine its length and inclinations with the reference planes.

19. Three guy ropes *AB*, *CD* and *EF* are tied at points *A*, *C* and *E* on a vertical post 15 m long at heights of 14 m, 12 m and 10 m respectively from the ground. The lower ends of the ropes are tied to hooks at points *B*, *D* and *F* on the ground level. If the points *B*, *D* and *F* lie at the corners of an equilateral triangle of 9 m long sides and if the post is situated at the centre of this triangle, determine graphically the length of each rope and its inclination with the ground. Assume the thickness of the post and the ropes to be equal to that of a line.

20. A line *AB*, 80 mm long, makes an angle of 60° with the H.P. and lies in an auxiliary vertical plane (A.V.P.), which makes an angle of 45° with the V.P. Its end *A* is 10 mm away from both the H.P. and the V.P.

    Draw the projections of *AB* and determine (i) its true inclination with the V.P. and (ii) its traces.

21. A line *PQ* is 75 mm long and lies in an auxiliary inclined plane (A.I.P.) which makes an angle of 45° with the H.P. The front view of the line measures 55 mm and the end *P* is in the V.P. and 20 mm above the H.P.

    Draw the projections of *PQ* and find (i) its inclinations with both the planes and (ii) its traces.

22. A line *AB*, 80 mm long, makes an angle of 30° with the V.P. and lies in a plane perpendicular to both the H.P. and the V.P. Its end *A* is in the H.P. and the end *B* is in the V.P. Draw its projections and show its traces.

23. The front view of a line makes an angle of 30° with xy. The H.T. of the line is 45 mm in front of the V.P., while its V.T. is 30 mm below the H.P. One end of the line is 10 mm above the H.P. and the other end is 100 mm in front of the V.P.

    Draw the projections of the line and determine (i) its true length, and (ii) its inclinations with the H.P. and the V.P.

24. A room is 6 m × 5 m × 3.5 m high. An electric bracket light is above the centre of the longer wall and 1 m below the ceiling. The bulb is 0.3 m away from the wall. The switch for the light is on an adjacent wall, 1.5 m above the floor and 1 m away from the other longer wall. Find graphically the shortest distance between the bulb and the switch.

25. Three lines *oa*, *ob* and *oc* are respectively 25 mm, 45 mm and 65 mm long, each making 120° angles with the other two and the shortest line being vertical. The figure is the top view of the three rods *OA*, *OB* and *OC* whose ends *A*, *B* and *C* are on the ground, while *O* is 100 mm above it. Draw the front view and determine the length of each rod and its inclination with the ground.

26. The projectors of the ends of a line *PQ* are 90 mm apart. *P* is 20 mm above the H.P. while *Q* is 45 mm behind the V.P. The H.T. and the V.T. of the line coincide with each other on *xy*, between the two end projectors and 35 mm away from the projector of the end *P*. Draw the projections of *PQ* and determine its true length and inclinations with the two planes.

27. A person on the top of a tower 30 m high, which rises from a horizontal plane, observes the angles of depression (below the horizon) of two objects *H* and *K* on the plane to be 15° and 25°, the direction of *H* and *K* from the tower being due north and due west respectively. Draw the top view to a scale of 1 mm = 0.5 m showing the relative positions of the person and the two objects. Measure and state in metres the distance between *H* and *K*.

28. Two pegs A and B are fixed in each of the two adjacent side walls (of a rectangular room) which meet in a corner. Peg A is 1.5 m above the floor, 1.2 m from the side wall and is protruding 0.3 m from the wall. Peg B is 2 m above the floor, 1 m from the other side wall and is also protruding 0.3 m from the wall. Find the distance between the ends of the pegs.
29. Two objects A and B, 10 m above and 7 m below the ground level respectively, are observed from the top of a tower 35 m high from the ground. Both the objects make an angle of depression of 45° with the horizon. The horizontal distance between A and B is 20 m. Draw to scale 1:250, the projections of the objects and the tower and find (a) the true distance between A and B, and (b) the angle of depression of another object C situated on the ground midway between A and B.
30. A room measures 8 m long, 5 m wide and 4 m high. An electric point hangs in the centre of the ceiling and 1 m below it. A thin straight wire connects the point to a switch kept in one of the corners of the room and 2 m above the floor. Draw the projections of the wire, and find the length of the wire and its slope-angle with the floor.
31. A rectangular tank 4 m high is strengthened by four stay rods one at each corner, connecting the top corner to a point in the bottom 0.7 m and 1.2 m from the sides of the tank. Find graphically the length of the rod required and the angle it makes with the surface of the tank.
32. Three vertical poles AB, CD and EF are respectively 5, 8 and 12 metres long. Their ends B, D and F are on the ground and lie at the corners of an equilateral triangle of 10 metres long sides. Determine graphically the distance between the top ends of the poles, viz. AC, CE and EA.
33. The front view of a line AB measures 70 mm and makes an angle of 45° with xy. A is in the H.P. and the V.T. of the line is 15 mm below the H.P. The line is inclined at 30° to the V.P. Draw the projections of AB, and find its true length, inclination with the H.P. and its H.T.
34. A line AB measures 100 mm. The projectors through its V.T. and the end A are 40 mm apart. The point A is 30 mm below the H.P. and 20 mm behind the V.P. The V.T. is 10 mm above the H.P. Draw the projections of the line and determine its H.T. and inclinations with the H.P. and the V.P.
35. A horizontal wooden platform is 3.5 m long and 2 m wide. It is suspended from a hook by means of chains attached at its four corners. The hook is situated vertically above the centre of the platform and at a distance of 5 m above it. Determine graphically the length of each chain and the angle which it makes with the platform. Assume the thickness of the platform and the chain to be equal to that of a line. Scale: 10 mm = 0.5 m.
36. A picture frame 2 m wide and 1 m high is to be fixed on a wall railing by two straight wires attached to the top corners. The frame is to make an angle of 40° with the wall and the wires are to be fixed to a hook on the wall on the centre line of the frame and 1.5 m above the railing. Find the length of the wires and the angle between them.
37. The top view of line AB measures 60 mm and inclined to reference line at 60°. The end point A is 15 mm above the H.P. and 30 mm infront of the V.P. Draw the projections of the line when it is inclined at 45° to the H.P. and is situated in the first quadrant. Find true length and inclination of the line with the V.P. and traces.